· Lecture 3. Interaction and entanglement

· For a single particle,

cyclotron coordinates $\leftarrow [\tilde{R}^a, \tilde{R}^b] = i\epsilon^{ab} l_B^2$ -> kinetic energy Landau levels

$\Rightarrow \hat{a}, \hat{a}^+$

$[\bar{R}^a, \bar{R}^b] = -i\epsilon^{ab} l_B^2$ -> guiding center coordinates

$\Rightarrow \hat{b}, \hat{b}^+$

$$\underline{\underline{[\tilde{R}, \bar{R}] = 0}} \qquad [\hat{a}, \hat{b}] = [\hat{a}, \hat{b}^+] = 0$$

· Let $|n\rangle$ be an eigenstate of $\hat{H}_{\text{kintic}}(\tilde{R})$

$$\hat{H}_{\text{kintic}}|n\rangle = \hbar\omega_c(n+\tfrac{1}{2})|n\rangle$$

$\hat{b}|n\rangle$, $\hat{b}^+|n\rangle$ are eigenstates of $\hat{H}_{\text{kintic}}$ with the same energy

· We need two integers to fully index an eigenstate of $\hat{H}_{\text{kintic}}$

$B=0$

Plane: $|k_x, k_y\rangle$

$B>0$

$|m,n\rangle$

$\hat{b}^+\hat{b}|m,n\rangle = m|m,n\rangle$

$\hat{a}^+\hat{a}|m,n\rangle = n|m,n\rangle$

$m = 0, 1, \ldots, N_\phi - 1$

$m = 0, 1,$

$$N_0 = \frac{A}{2\pi l_B^2}$$

Sphere: $|m, l\rangle$

$l = 0, 1, \dots$

$m = -l, -l+1, \dots l$

$|m, l\rangle$ 2S is the total flux

$l = S, S+1, \dots$

$m = -l, \dots l$

$\underbrace{2S}_{\text{magnetic flux}} + \underbrace{2n+1}_{\text{Spherical curvature}}$ states

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- First quantised wavefunctions:

  - On the plane, we pick a symmetric gauge:

$$A_x = -\frac{1}{2} B y, \quad A_y = \frac{1}{2} B x,$$

$$\partial_x A_y - \partial_y A_x = B$$

$$z = x - iy$$

$$z^* = x + iy$$

$$\underline{\underline{\hat{a}}} = \frac{1}{\sqrt{2}} \left(\frac{z}{2} + 2 \frac{\partial}{\partial z^*} \right)$$

$$\hat{a}^+ = \frac{1}{\sqrt{2}} \left(\frac{z^*}{2} - 2 \frac{\partial}{\partial z} \right)$$

$$\underline{\underline{\hat{b}}} = \frac{1}{\sqrt{2}} \left(\frac{z^*}{2} + 2 \frac{\partial}{\partial z} \right)$$

$$\hat{b}^+ = \frac{1}{\sqrt{2}} \left(\frac{z}{2} - 2 \frac{\partial}{\partial z^*} \right)$$

$$\underline{\underline{L_z}} = x P_y - y P_x$$

$$\sim \underline{\underline{\hat{a}^+ \hat{a}}} - \underline{\underline{\hat{b}^+ \hat{b}}}$$

For lowest Landau level (LLL), $n = 0$

$$\psi_m(z,z^*) = \langle z,z^* | m,0\rangle = \langle z,z^* | \frac{1}{\sqrt{m!}}(b^\dagger)^m | 0,0\rangle$$

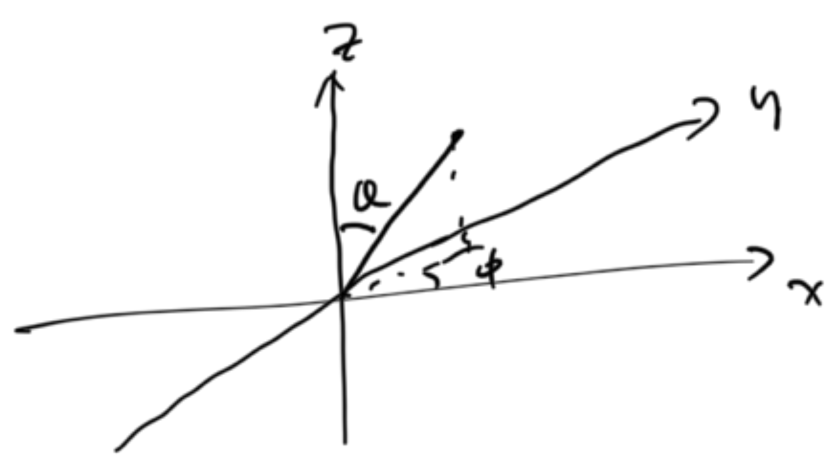
$$= \frac{1}{\sqrt{2\pi 2^m \cdot m!}} \underline{z}^m \cdot e^{-\frac{1}{4}zz^*}$$

A general LLL wavefunction is given by

$$\underbrace{f(z)}_{\text{holomorphic function}} e^{-\frac{1}{4}zz^*}$$

On the sphere, we pick a gauge:

$$\vec{A} = \left(\frac{\hbar \cdot S}{eR}\cot\theta + \frac{1}{\sin\theta}\right)\hat{\phi}$$



For LLL wavefunction, we define

$$u = \cos(\theta/2)\, e^{-i\phi/2}$$
$$v = \sin(\theta/2)\, e^{+i\phi/2}$$
(spinor coordinates)

$$\psi_{m,l=s}(\theta,\phi) = \left[\frac{2s+1}{4\pi}\binom{2s}{s-m}\right]^{\frac{1}{2}} (-1)^{s-m} \cdot u^{s+m} v^{s-m}$$

Let $m' = s-m$, $m' = 0, 1, \ldots 2s$

$$z = \frac{v}{u} = \tan\theta/2\; e^{i\phi}$$

$$\psi_{m', l=s} \sim z^{m'} \cdot \left(\frac{1}{1+zz^*}\right)^s$$

Single particle state.

related to $F_n(q^2)$

Disk:

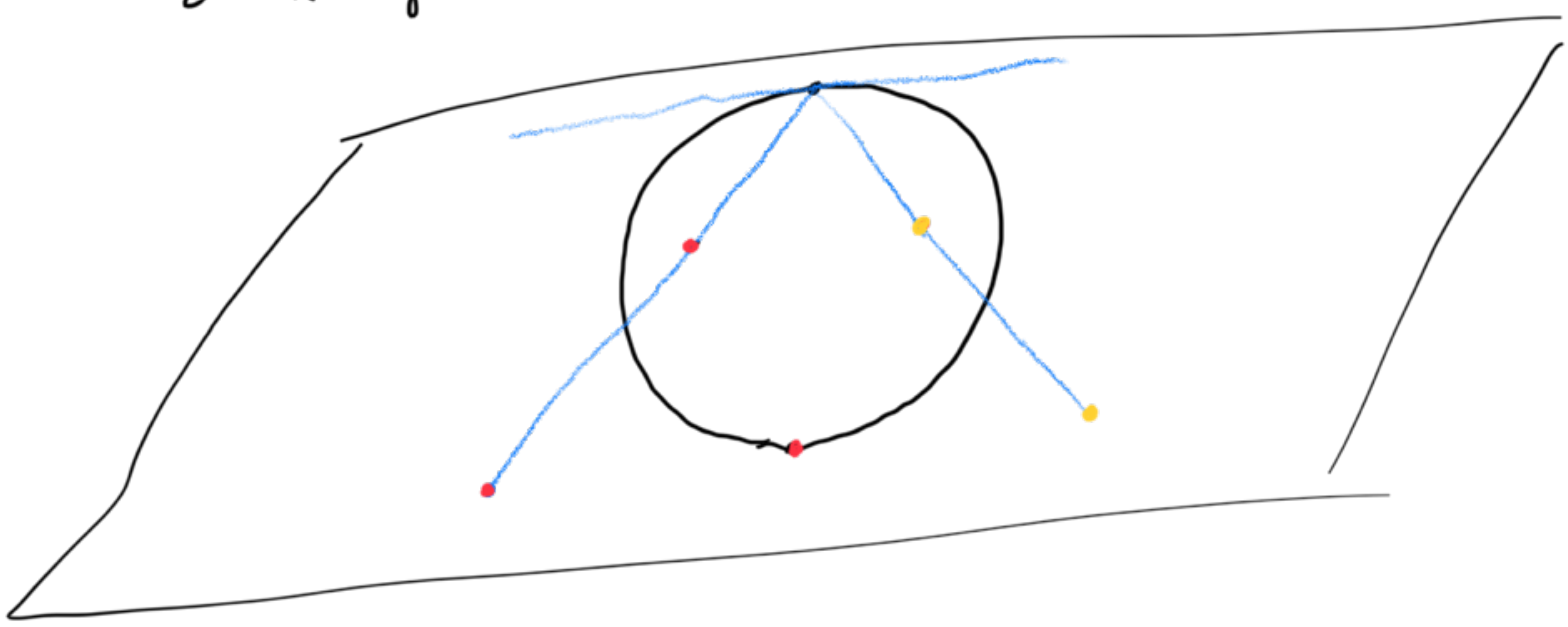
$$\underline{\underline{z^m}} \cdot e^{-\frac{1}{4} z z^*}$$

$z = x - iy$

Sphere

$$\underline{\underline{z^m}} \left(\frac{1}{1+zz^*}\right)^s$$

$z = \tan\theta/_2 \, e^{i\phi}$



. Interaction within a single Landau level.

$$\hat{H}_{int} = \int d^2r_1 d^2r_2 \, V(r_1 - r_2) \, \rho(r_1) \rho(r_2)$$

$$= \int d^2q \, V_q \, \rho_q \, \rho_{-q}$$

$$\rho(r) = \sum_i \delta(r_i - r) \rightarrow \rho_q = \sum_i e^{iq \cdot r_i}$$

$$= \sum_i e^{iq \tilde{R}_i} e^{iq R_i}$$

For Coulomb interaction

$$V(r_1 - r_2) = \frac{1}{|r_1 - r_2|}, \quad V_q = \frac{1}{|q|}$$

. Let us fix the Landau level index.

. fixed

$|m,n\rangle = |m\rangle$ $\quad$ $\underline{\underline{n}}$ is q...

A two-body state:

$$|\underline{\underline{m_1}}, \underline{\underline{m_2}}\rangle = C^\dagger_{m_1} C^\dagger_{m_2} |vac\rangle = \underline{\underline{C^\dagger_{m_1}}} \underline{\underline{C^\dagger_{m_2}}} |0\rangle$$

second-quantised creation operator

· The matrix elements of the two-body Hamiltonian

$$V^{n_1 n_2}_{m_1, m_2} = \langle n_1, n_2| \hat{H}_{int} |m_1, m_2\rangle$$

$$= \underline{\int d^2q\, V_q \langle n_1, n_2| \rho_q \rho_{-q} |m_1, m_2\rangle}$$

$$\hat{H}_{int} = \sum_{\substack{n_1 n_2 \\ m_1 m_2}} \underline{\underline{V^{n_1 n_2}_{m_1 m_2}}} C^\dagger_{n_1} C^\dagger_{n_2} C_{m_1} C_{m_2}$$

$\hookrightarrow$ second quantized form

$$\hat{S}_q = \underline{\underline{\sum_{i \neq j} e^{iq(r_i - r_j)}}} = \rho_q \rho_{-q} + \underline{\underline{\text{constant}}}$$

$$= \sum_{i \neq j} \underline{\underline{e^{iq(\tilde{R}_i - \tilde{R}_j)}}} \underline{\underline{e^{iq(\bar{R}_i - \bar{R}_j)}}}$$

static structure factor

$$S_q = \langle \underline{n_1}, \underline{n_2}| \hat{S}_q |\underline{m_1}, \underline{m_2}\rangle$$

$$= \underline{\underline{\langle n_1, n_2|}} \sum_{i \neq j} e^{iq(\bar{R}_i - \bar{R}_j)} \underline{\underline{|m_1, m_2\rangle}}$$

$$\underline{\underline{\left(\langle n| e^{iq\tilde{R}} |n\rangle\right)^2}} \hookrightarrow F_n(q^2)$$

· $\tilde{R}^x = \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^\dagger)$ $\qquad$ $|n\rangle = \frac{1}{\sqrt{n!}}(\hat{a}^\dagger)^n|0\rangle$

$\tilde{R}^y = \frac{i}{\sqrt{2}}(\hat{a}^\dagger - \hat{a})$,

$q \cdot \tilde{R} \sim \bar{q}\hat{a}^\dagger + \bar{q}^*\hat{a}$, $\quad \bar{q} = q_x + iq_y$

· Baker-Campbell-Hausdorff formula

This depends on the orbital wave function flat band in the q

$$F_n(q^2) = e^{-\frac{1}{4}q^2} \cdot L_n(q^2/2)$$

$\hookrightarrow$ $\boxed{\text{Laguerre polynomial}}$

$$L_n^{\alpha}(x) = \sum_{i=1}^{n} (-1)^i \cdot \binom{n+\alpha}{n-i} \cdot \frac{x^i}{i!}$$

$$L_n^{0}(x) = L_n(x)$$

$$V_{m_1 m_2}^{n_1 n_2} = \int d^2q \; V_q \cdot \left(F_n(q^2)\right)^2 \langle n_1, n_2 | \, e^{iq(\bar{R}_1 - \bar{R}_2)} \, | m_1, m_2 \rangle$$

Let $\bar{R}^a_{1,2} = \frac{1}{2}(\bar{R}^a_1 + \bar{R}^a_2) \rightarrow \hat{b}^{+}_{1,2}, \hat{b}_{1,2}$

$\bar{R}^a_{12} = \frac{1}{2}(\bar{R}^a_1 - \bar{R}^a_2) \rightarrow \hat{b}^{+}_{12}, \hat{b}_{12}$

center of mass

$\hookrightarrow$ relation

$$|m_1, m_2\rangle \; \frac{1}{\sqrt{m_1! m_2!}} (\hat{b}^{+}_1)^{m_1} (\hat{b}^{+}_2)^{m_2} |0\rangle$$

$$\sim (\hat{b}^{+}_{1,2} + \hat{b}^{+}_{12})^{m_1} \cdot (\hat{b}^{+}_{1,2} - \hat{b}^{+}_{12})^{m_2} |0\rangle$$

$$\sim \binom{m_1}{k_1}\binom{m_2}{k_2} (\hat{b}^{+}_{1,2})^{k_1+k_2} (\hat{b}^{+}_{12})^{m_1+m_2-k_1-k_2} |0\rangle$$

$$V_{m_1 m_2}^{n_1 n_2} = \sum_{\substack{k_1 k_2 \\ k_3 k_4}} N(k_1, k_2, k_3, k_4, m_1, m_2, n_1, n_2) \cdot \delta(k_1 + k_2 - k_3 - k_4)$$

$\hookrightarrow$ center of mass

$\cdot$ relative part in terms of Laguerre polynomial

· The general model for two-body interaction.

$$\bar{H}_{int} = \int d^2q\, \bar{V}_q\, \bar{\rho}_q \bar{\rho}_{-q}$$ → this is not a quadratic Hamiltonian

microscopic details

$$\bar{V}_q = V_q \cdot \left(F_n(q^2)\right)^2$$ ✓✓

$$\bar{\rho}_q = \sum_i e^{iq\bar{R}_i}$$ ✓✓

$$[\bar{\rho}_{q_1}, \bar{\rho}_{q_2}] = 2i \sin\frac{q_1 \times q_2}{2}\, \bar{\rho}_{q_1+q_2}$$ ✓✓

→ Girvin-Macdonald-Platzmann algebra (W_∞ algebra)

· $$\hat{H}_{int} = \int d^2q\, \bar{V}_q\, \bar{\rho}_q \bar{\rho}_{-q}$$

· Special case.

$$\bar{V}_q = V_q \left(F_n(q^2)\right)^2 = L_1(q^2)\, e^{-\frac{1}{2}q^2} = V_1(q^2)$$

$$H_1 = \int d^2q\, V_1(q^2)\, \bar{\rho}_q \bar{\rho}_{-q}$$ → 1st Haldane pseudopotential

$$\hat{b}_{1,2} = \frac{1}{\sqrt{2}}(\hat{b}_1 + \hat{b}_2)$$ ✓

$$\hat{b}_{12} = \frac{1}{\sqrt{2}}(\hat{b}_1 - \hat{b}_2)$$ ✓

$$|m, M\rangle = \frac{1}{\sqrt{m!M!}} \left(\hat{b}_{12}^\dagger\right)^m \left(\hat{b}_{1,2}^\dagger\right)^M |0\rangle$$

$$= \sum N(m, M, k_1, k_2)\, |k_1, k_2\rangle$$

$$\sim \sum_{k_1, k_2} N(m, M, k_1, k_2) \, | k_1, k_2 \rangle$$

$$(\hat{b}_1^{+})^{k_1} (\hat{b}_2^{+})^{k_2} | 0 \rangle$$

$$\langle m', M' | H_1 | m, M \rangle = \underline{\delta_{M', M} \, \delta_{m, 1} \, \delta_{m', 1}}$$

(Orthogonality condition for Laguerre polynomials)

$$| m_1, m_2 \rangle = \frac{1}{\sqrt{m_1! m_2!}} (\hat{b}_1^{+})^{m_1} (\hat{b}_2^{+})^{m_2} | 0 \rangle$$

$$= \sum_{m, M} \underline{U_{m, M}^{m_1 m_2}} \, | m, M \rangle$$

$$V_{m_1 m_2}^{n_1 n_2} = \sum_{\substack{m, M \\ n, N}} U^{* \, n_1 n_2}_{n, N} \, U^{m_1 m_2}_{m, M} \, \langle n, N | H_1 | m, M \rangle$$

$$= \sum_{M} U^{* \, n_1 n_2}_{1, M} \, U^{m_1 m_2}_{1, M}$$

- If any two particles have relative angular momentum greater than 1, then its energy is zero.

$$(\hat{b}_{1,2}^{+})^{M} (\hat{b}_{12}^{+})^{m} | 0 \rangle = | m, M \rangle$$

$$\psi(z_1, z_2) \sim (z_1 + z_2)^M (z_1 - z_2)^m$$ → relative angular momentum

→ two particles, all states with

$(z_1 + z_2)^M (z_1 - z_2)^{m > 1}$ has zero energy

$$m = 3, 5, 7, \cdots$$

For $m=3$, highest density state

$$(z_1 - z_2)^3 = z_1^3 - z_2^3 - 3(z_1^2 z_2 - z_1 z_2^2)$$

$$\sim \quad |1001\rangle \quad ✓$$

$$-3\,|0110\rangle \quad ✓$$

→ Three particles

The highest density state:

$$\underline{(z_1 - z_2)^3}\ \underline{(z_2 - z_3)^3 (z_3 - z_1)^3} \quad ✓✓$$

$$\sim \quad |1001001\rangle \quad ✓$$

$$-3\,|0110001\rangle \quad ✓$$

$$-3\,|1000110\rangle \quad ✓$$

$$+6\,|0101010\rangle \quad ✓$$

$$-15\,|0011100\rangle \quad ✓$$

→ For N particles

$$\prod_{i<j} (z_i - z_j)^3 \ \underline{\underline{e^{-\frac{1}{4}\sum_i |z_i|^2}}} \rightarrow \text{Laughlin state}$$

$\sim \quad | 100100 | \cdots \; 00 | \cdots$

$$= J^{\alpha=-2}_{\lambda = 1001001 \cdots 01001} \quad \text{(Jack polynomial)}$$

· Model Hamiltonian as a projector

$$H_n = \int d^2q \; V_n(q)\, \hat\rho_q \hat\rho_{-q} \qquad V_n(q) = L_n(q^2)\, e^{-\frac{1}{2}q^2}$$

$$= \int d^2q \; V_n(q) \sum_{i \neq j} e^{i q \cdot (\bar R_i - \bar R_j)} + \text{const.}$$

$$= \sum_{\substack{m_1, m_2 \\ n_1, n_2}} U^{n_1 n_2}_{m_1 m_2}\, c^\dagger_{n_1} c^\dagger_{n_2} c_{m_1} c_{m_2}$$

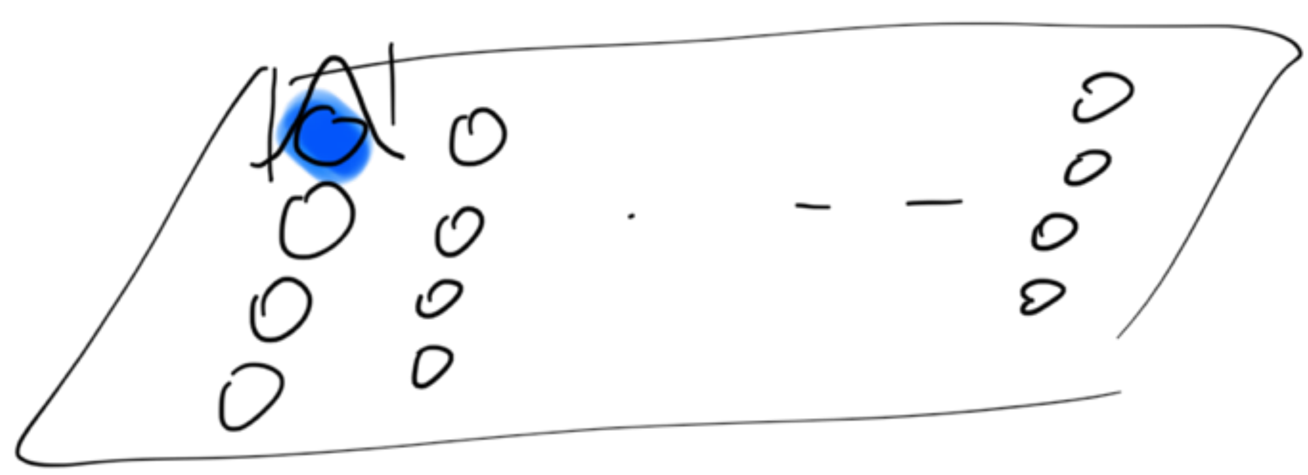
$$\boxed{V_n} = \sum_{M} |n, M\rangle \langle n, M| \qquad \text{for two particles}$$

$$|n, M\rangle \sim (\hat b^\dagger_{12})^n (\hat b^\dagger_{1,2})^M |0\rangle$$

$$V_n = \sum_{\vec X} |n, 0\rangle_{\vec X} \; {}_{\vec X}\langle n, 0|$$

$$|n, 0\rangle_{\vec X} = e^{i \cdot \vec X \cdot \bar R} |n, 0\rangle$$

→ Lattice points of the Von Neumann lattice



· Anisotropic generalization

$$\bar{R}^x, \bar{R}^y \to \hat{b}, \hat{b}^+$$

$$\to \tilde{b} = \cosh\theta\, \hat{b} + \sinh\theta\, e^{i\phi}\, \hat{b}^+ \Rightarrow \bar{R}_x', \bar{R}_y'$$

$$\tilde{b}^+ = \cdots$$

(Bogoliubov transformation)

$$\sum_{i \neq j} e^{iq(\bar{R}_1 - \bar{R}_2)} \to \sum_{i\neq j} e^{iq'(\bar{R}_1' - \bar{R}_2')}$$

$$\Rightarrow V_n(q) = L_n(|q|^2)\, e^{-\frac{1}{2}|q|^2}$$

Same energy spectrum, same physics

$$\Rightarrow L_n(|q|_g^2)\, e^{-\frac{1}{2}|q|_g^2} = V_n(q_g)$$

$$|q|_g^2 = g^{ab} q_a q_b$$

↳ unimodular metric

The Laughlin state, $(n=1)$

$$\Psi_L \sim \prod_{i<j} \left(\hat{b}_i^+ - \hat{b}_j^+\right)^3 |0\rangle \qquad \hat{b}|0\rangle = 0$$

very small overlap

$$\to \prod_{i<j} \left(\tilde{b}_i^+ - \tilde{b}_j^+\right)^3 |\tilde{0}\rangle \qquad \tilde{b}|\tilde{0}\rangle = 0$$

Example for two particles

$$(z_1 - z_2)^3\, e^{-\frac{1}{4}(|z_1|^2 + |z_2|^2)}$$

$$\to (\tilde{z}_1 - \tilde{z}_2)(\tilde{z}_1 - \tilde{z}_2 + \alpha)(\tilde{z}_1 - \tilde{z}_2 - \alpha^*)$$

$$-t(z_1^2 + z_2^2) \cdot e^{-\frac{1}{4}(|z_1|^2 + |z_2|^2)}$$

$\cdot\, e$

$$\tilde{z}_i = \frac{e^{-i\phi}}{\cosh\theta} z_i, \quad t = \tanh\theta\, e^{-2i\phi}, \quad \alpha = \sqrt{3}\cdot t$$

- First quantized wavefunction is "bad" for single LL physics (FQH physics)

- Rotational symmetry is not important for FQH

- Generalization to spatial variation.

The general two-body interaction

$$H = \int d^2q_1 d^2q_2\, \bar{V}_{q_1 q_2}\, \bar{\rho}_{q_1}\, \bar{\rho}_{q_2}$$

$$= \int d^2q_1 d^2q_2\, \bar{V}_{q_1 q_2} \sum_{i \neq j} e^{iq'(\bar{R}_i + \bar{R}_j)} \cdot e^{iq(\bar{R}_i - \bar{R}_j)}$$

$$f(q') = \sum_{M} C_M L_M(q^2) e^{-\frac{1}{2}q^2}$$

$q' + q \sim q_1$

$q' - q \sim q_2$

- $$H_n = \int d^2q'\, d^2q\, f(q') V_n(q) \cdot \sum_{i \neq j} e^{iq'(\bar{R}_i + \bar{R}_j)} e^{iq(\bar{R}_i - \bar{R}_j)}$$

$$= \sum_M C_M |n, M\rangle \langle n, M|$$

$$= \sum_{\vec{X}'} |n, 0\rangle_{\vec{X}'}\ {}_{\vec{X}'}\langle n, 0|$$

↳ deformed Von Neuman lattice

Different spectrum (tune the gap), but ground state and quasihole states have exact zero energy.

- Translational invariance is not important for

FQHE

· Relevant to FCI

$\Rightarrow$ Generalization to few-body interaction

· Many-body wavefunction and entanglement

· The monomials

$$Asy(z_1^{m_1} z_2^{m_2} \cdots z_N^{m_N}) =$$

$$\sim C^{\dagger}_{m_1} C^{\dagger}_{m_2} \cdots C^{\dagger}_{m_N} |vac\rangle$$

$$\sim |\underset{m_0}{0}0 \cdots \underset{m_1}{1}\, 00 \cdots \underset{m_2}{1}\, 0 \cdots \underset{m_3}{1} \cdots \cdots 0\, \underset{m_N}{1}\, 00\rangle$$

$$= |m_1, m_2, \cdots m_N\rangle \checkmark$$

· A monomial is a product state in orbital basis

· Example,

$$|\psi\rangle \sim Asy(z_1^0 z_2^3 z_3^6 z_4^{10})$$

$$= |0, 3, 6, 10\rangle$$

$N_0 = 11$

$$= |\,|00|\,00\ldots\rangle$$

A B

↳ orbital cut

disk

sphere

$$|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle \quad \rightarrow \text{ product state}$$

$$|\psi_A\rangle = |100|0\rangle, \qquad |\psi_B\rangle = |0|0001\rangle$$

. The density matrix

$$\hat{\rho} = |\psi\rangle\langle\psi| \quad \rightarrow \text{ a projection operator for a pure state}$$

For an observable

$$\tilde{O} = \langle\psi|\hat{O}|\psi\rangle$$

$$= \sum_i \langle\psi|\psi_i\rangle\langle\psi_i|\hat{O}|\psi_i\rangle\langle\psi_i|\psi\rangle$$

↳ eigenstates of $\hat{O}$

$$= \sum_i \langle\psi_i|\psi\rangle\langle\psi|\psi_i\rangle O_i$$

$$O_i = \langle\psi_i|\hat{O}|\psi_i\rangle$$

$$= \sum_i \langle\psi_i|\hat{\rho}\hat{O}|\psi_i\rangle$$

$$= \mathrm{Tr}(\hat{\rho}\hat{O})$$

$\Rightarrow =$, basis independent

$\hat{\rho}$ can represent a pure quantum state, or a mixed state

e.g a thermal system

$$\hat{\rho} = \sum_i e^{-\beta E_i} |\psi_i\rangle\langle\psi_i|$$

- The reduced density matrix

  - If we divide the system into two parts A and B

$$\hat{\rho}_A = \sum_i \langle\psi_{i,B}| \hat{\rho} |\psi_{i,B}\rangle = \mathrm{Tr}_B \hat{\rho}$$

$|\psi_{i,B}\rangle$ spans the Hilbert space of B

If $\hat{\rho} = |\psi\rangle\langle\psi|$

$$|\psi\rangle = \sum_{i,j} C_{ij} |\psi_{i,A}\rangle \otimes |\psi_{j,B}\rangle$$

Then

$$\hat{\rho}_A = \mathrm{Tr}_B \rho = \sum_k \langle\psi_{k,B}| \hat{\rho} |\psi_{k,B}\rangle$$

$$= \sum_{i,j,k} C_{ij}^* C_{jk} |\psi_{i,A}\rangle\langle\psi_{k,A}|$$

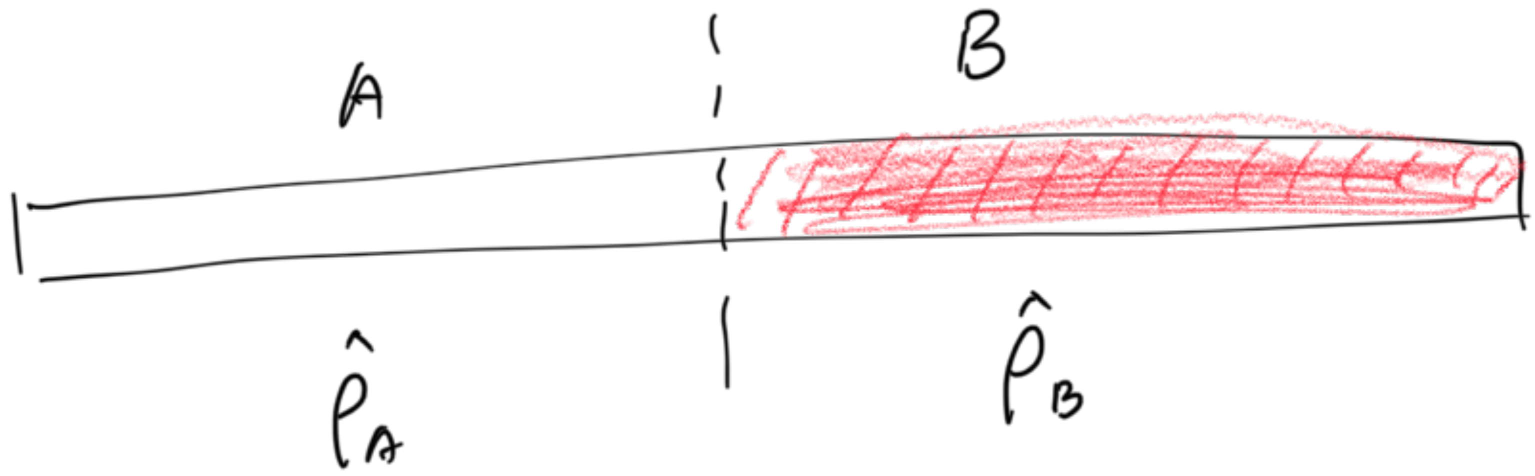
↳ mixed state

For product state: $\hat{\rho}_A = |\psi_A\rangle \otimes \langle\psi_A|$

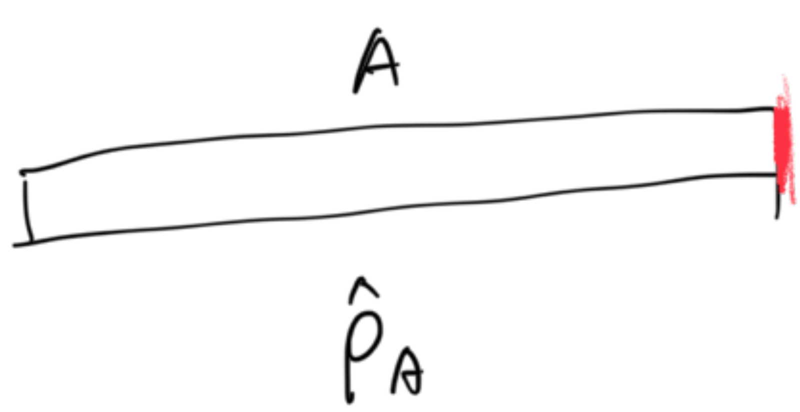
· If $\hat{O}$ only does measurements in A

$$\tilde{O} = \langle\psi|\hat{O}|\psi\rangle = \mathrm{Tr}\langle\hat{\rho}\hat{O}\rangle$$

$$= \mathrm{Tr}_A\langle\hat{\rho}_A\hat{O}\rangle$$



After tracing out subspace B,



→ virtual boundary containing entanglement with B (an event horizon of black hole)

· The entanglement entropy of $\hat{\rho}_A$

$$\hat{\rho}_A = \sum_{i,j} \lambda_{ij} |\psi_i\rangle\langle\psi_j|$$

$$= \sum_k \bar{\lambda}_k |\bar{\psi}_k\rangle\langle\bar{\psi}_k|$$

$$\lambda_{ij} = U_{ik}\bar{\lambda}_k V_{kj}$$ → diagonalization or singular value decomposition

The entanglement entropy

$$S = -\sum \bar{\lambda}_k \log \bar{\lambda}_k = -\mathrm{Tr}_A(\hat{\rho}_A \log \hat{\rho}_A)$$

· For product state, $\hat{\rho}_A = |\psi_A\rangle \otimes \langle\psi_A|$

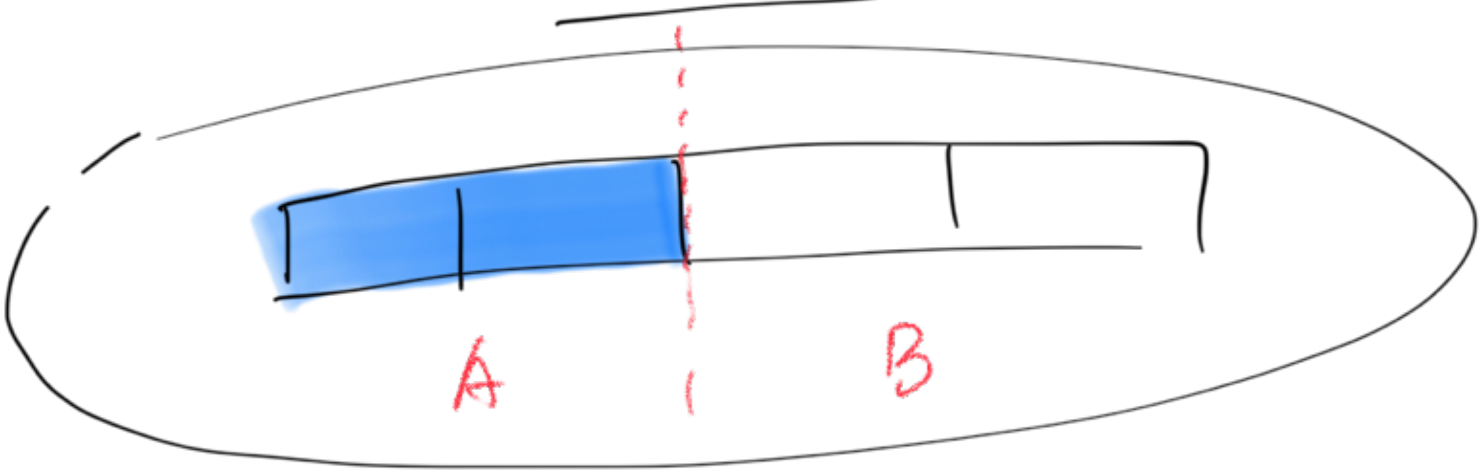
$$S_A = 0$$

· For Laughlin state.

$$|\psi\rangle \sim (z_1 - z_2)^3$$

$$\sim \underline{|1001\rangle} \quad \underline{-3|0110\rangle} \Rightarrow \underline{|\psi\rangle = \tfrac{1}{\sqrt{2}}\left(|1001\rangle - |0110\rangle\right)}$$

on sphere



$$|\psi\rangle = \frac{1}{\sqrt{2}}\underbrace{|10\rangle}_{|\psi_{1,A}\rangle}\otimes\underbrace{|01\rangle}_{|\psi_{1,B}\rangle} - \frac{1}{\sqrt{2}}\underbrace{|01\rangle}_{|\psi_{2,A}\rangle}\otimes\underbrace{|10\rangle}_{|\psi_{2,B}\rangle}$$

$$= \frac{1}{\sqrt{2}}\left(|\psi_{1A}\rangle|\psi_{1B}\rangle - |\psi_{2A}\rangle|\psi_{2B}\rangle\right)$$

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

$$\Downarrow$$

$$\hat{\rho}_A = \frac{1}{2}\left(|\psi_{1A}\rangle\langle\psi_{1A}| + |\psi_{2A}\rangle\langle\psi_{2A}|\right)$$

$$S_A = 2\times\left(-\frac{1}{2}\log\frac{1}{2}\right) = \log 2 > 0$$

· The full Hilbert space of A is

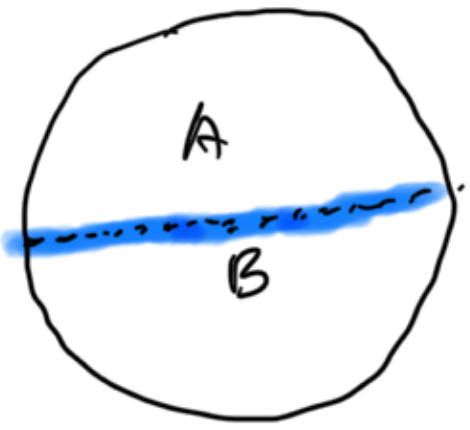
$|00\rangle$, $|10\rangle$, $|01\rangle$, $|11\rangle$

[redacted]

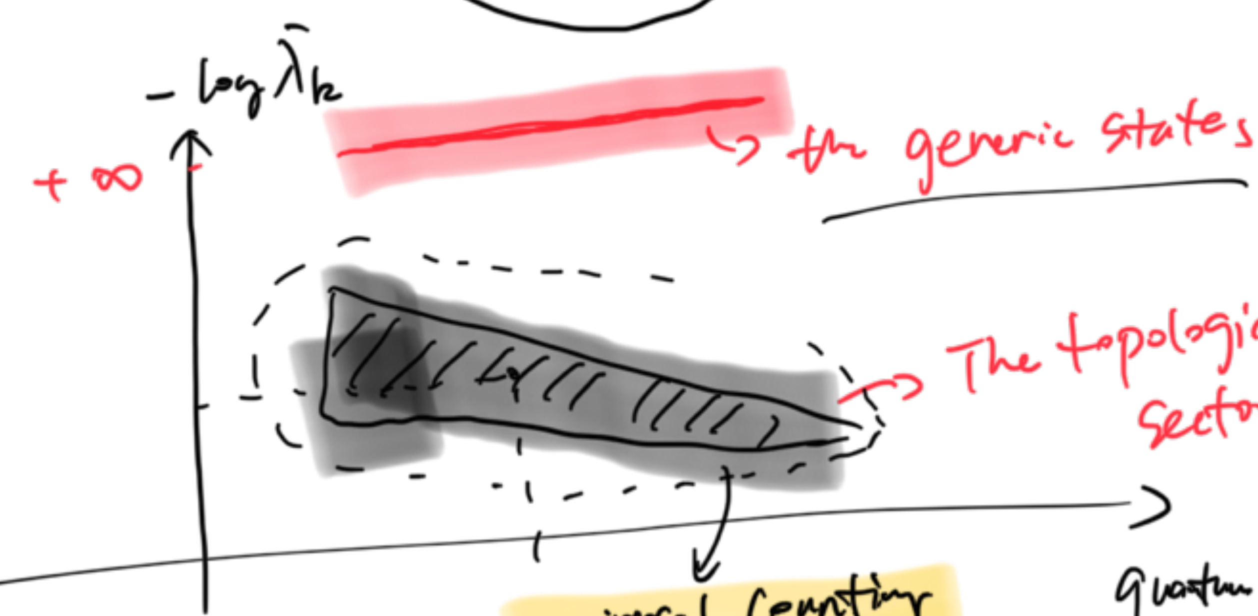
- The Laughlin state has missing states in $\hat{\rho}_A$

$|00\rangle$, $|11\rangle$

- In general for a topological FQH state



$$\hat{\rho}_A = \sum_k \lambda_k |\psi_{kA}\rangle\langle\psi_{kA}|$$



entanglement spectrum

area law

$S_A = La - \gamma$

topological entanglement entropy

universal counting of states

- Quantum entanglement $\Leftrightarrow$ ground state topology

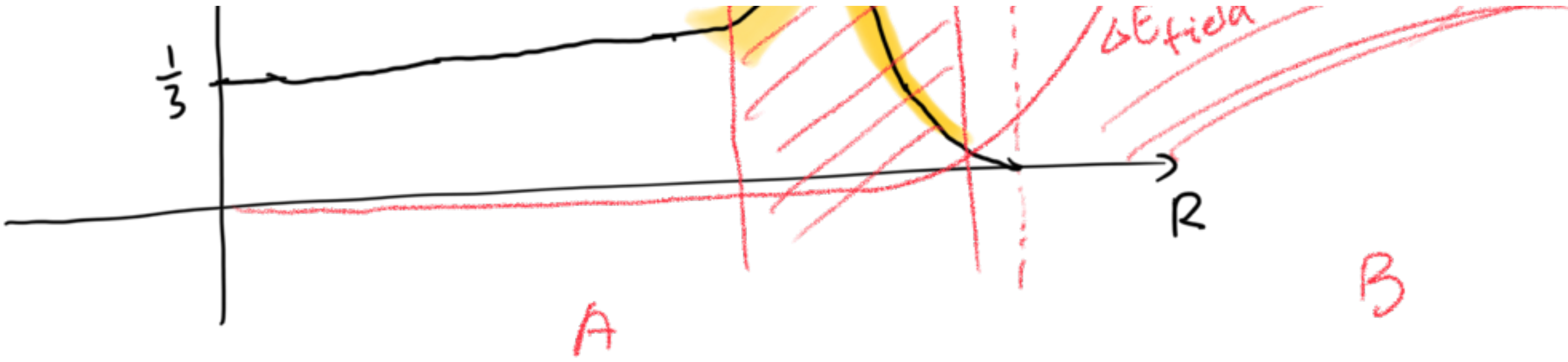
$\Leftrightarrow$ Hilbert space truncation

$\Leftrightarrow$ Conformal symmetry

universal for FQHE

$$F_1 \sim D \cdot \Delta E_{field} = F_2 \sim \bar{S} \cdot \Delta V \sim \bar{S} \cdot \Delta E_{field}$$

(D) dipole moment $\to$ Hall viscosity



$$\boxed{\hat r^a = \underbrace{\tilde R^a}_{\hat a,\,\hat a^\dagger} + \underbrace{\bar R^a}_{\hat b,\,\hat b^\dagger}}$$

Hall viscosity has contributions

cyclotron + guiding center