Lecture 5: Conformal Hilbert space I

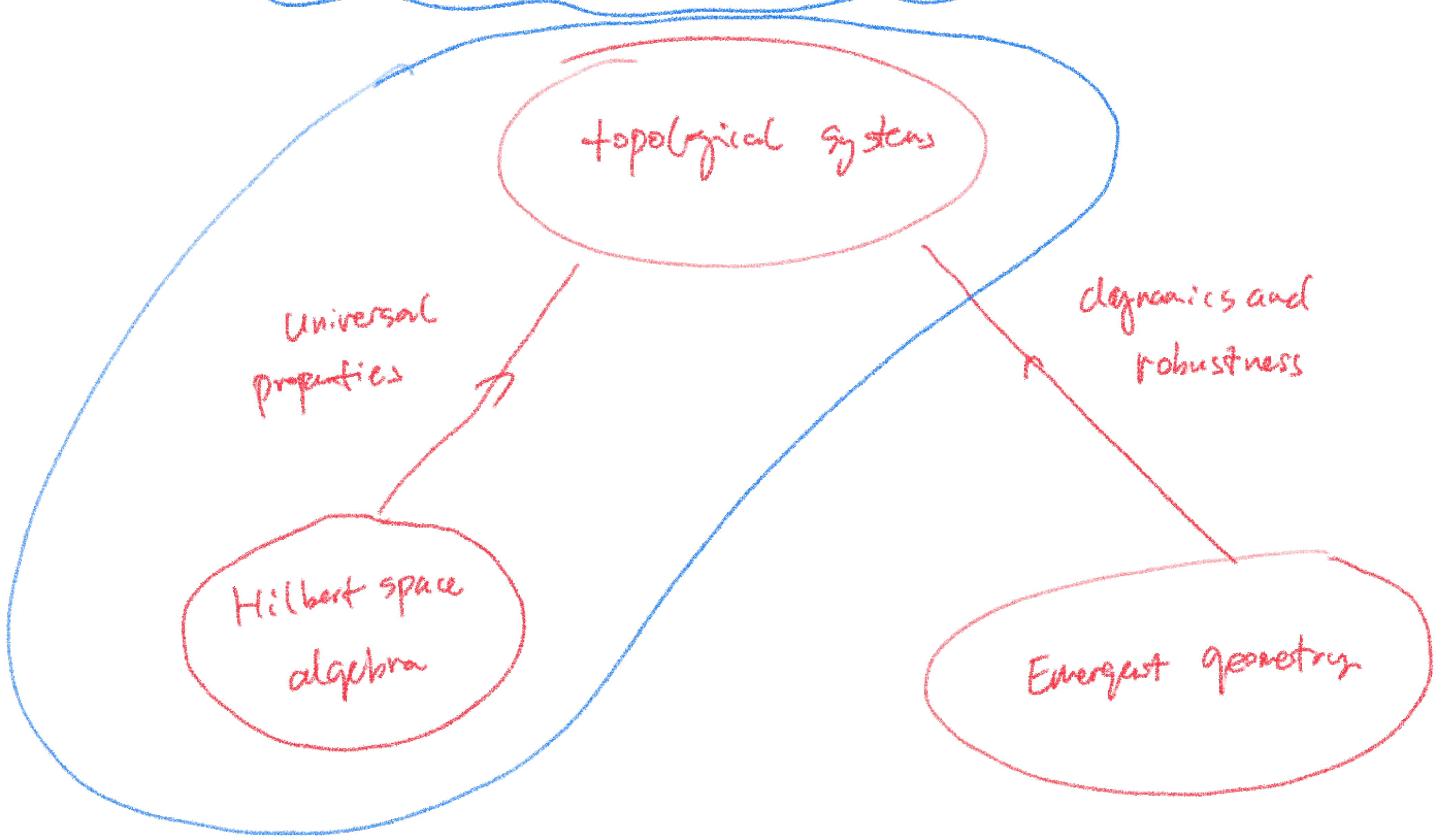
- What is the use of the LEC formalism
  - Simplicity for intuition and numerics
  - Potential for unifying CF theory and CFT / model Hamiltonian approach

    Jain state at $2/5$ PRB 100, 245303

    ... $2/9$ arXiv: 2201.00020

  - FQHE / IQHE from conformal Hilbert spaces

Topological properties of FQHE fundamentally emerge from Hilbert space truncation



- Full Hilbert space (B=0)

- Elementary particles are electr…
  - · Fermions, carrying a charge e. and spin $\frac{1}{2}$
  - · Effective mass m^* or Fermi velocity v_F^*
  - · point particles ($r < 10^{-22}$ m)

• A single Landau level (LL) $B \to \infty$

- A sub-Hilbert space
- Elementary particles are "quasiparticles"
  - Fermions, carrying a charge of e, and spin $\frac{1}{2}$
  - No dispersion, $m^* \to \infty$ or $v_F^* \to 0$
  - NOT point particles (occupy a finite area $\underbrace{2\pi \ell_B^2}$)

a loss of information due to LL projection

$$[\bar{R}^x, \bar{R}^y] = -i\ell_B^2$$

- A Hilbert space with conformal symmetry

↳ angle-preserving

- $P_\mu = -i\partial_\mu$ (translation)

$D = -i x^\mu \partial_\mu$ (dilation)

$L_{\mu\nu} = x_\mu P_\nu - x_\nu P_\mu$ ($\vec{r} \times \vec{p}$) (rotation)

$L^x = L_{yz}$
$L^y = L_{zx}$
$L^z = L_{xy}$

$K_\mu = -i(2x_\mu x^\nu \partial_\nu - x^2 \partial_\mu)$ (SCT)

there is a hidden metric

If we define $J_{\mu\nu} = L_{\mu\nu}$ $\mu, \nu = 1, 2, 3$ ($x\ y\ z$)

$J_{-1,0} = D$

$$J_{0,\mu} = \frac{1}{2}(P_\mu + K_\mu)$$

$$J_{-1,\mu} = \frac{1}{2}(P_\mu - K_\mu)$$

$$J_{ab} = -J_{ba}$$

$$\Rightarrow [J_{ab}, J_{cd}] = i(\eta_{ad} J_{bc} + \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac})$$

$$\eta_{ab} = \mathrm{diag}(-1, 1, \cdots 1)$$

Conformal group in d-dimension

$$\cong SO(d+1, 1) \qquad (-1, \underbrace{1, \cdots 1}_{d+1})$$

- for $d=2$, $SO(3,1)$ is the Lorentz group
- Conformal symmetry as a "glorified" version of $SU(2)$ algebra (rotation)

- In two-dimension, we are interested in local conformal symmetry
  - The Virasoro algebra

$$[\hat{L}_n, \hat{L}_m] = (n-m)\hat{L}_{m+n} + \frac{c}{12} n(n^2-1)\delta_{m+n,0}$$

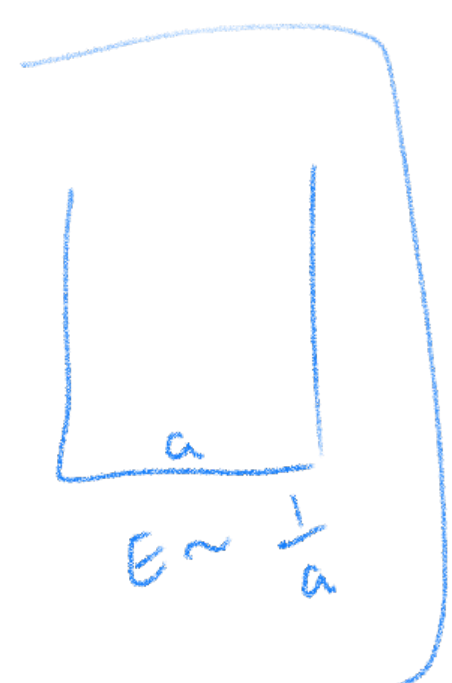
c → central charge

n, m are integers

a quantum anomaly

- quantization from macroscopic scale

"softly" break conformal symmetry



☆ The composition of conformal Hilbert space

- A highest density state (FQH ground state)

$$|\psi_{h_0}\rangle \to \hat{L}_0|\psi_{h_0}\rangle = h_0|\psi_{h_0}\rangle$$

$$\hat{L}_{n>0}|\psi_{h_0}\rangle = 0 \quad \text{(HW condition)}$$

present for non-Abelian FQH ← - A collection of highest weight states

$$\{|\psi_{h_i}\rangle\} \quad i = 1, 2, \ldots n_h \quad h_i > h_0$$

$$\to \hat{L}_0|\psi_{h_i}\rangle = h_i|\psi_{h_i}\rangle$$

$$\hat{L}_{n>0}|\psi_{h_i}\rangle = 0$$

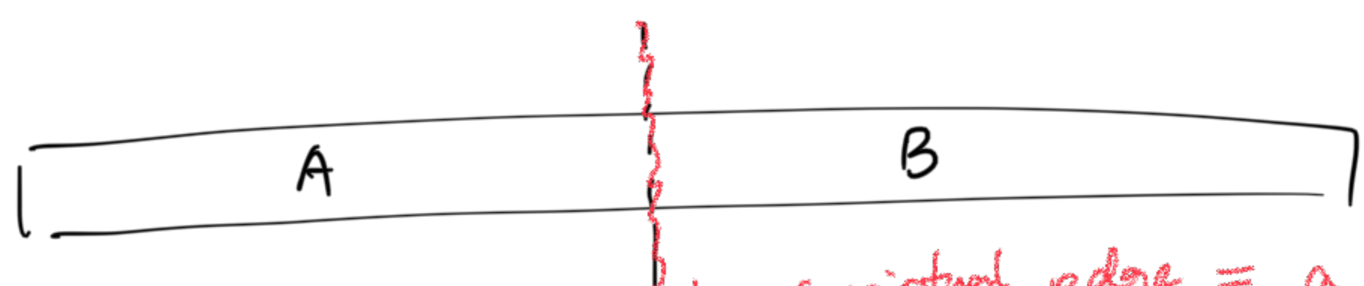
(FQH quasihole states)

- All other "secondary states"
  (quasihole states)

  - Generated by applying $\hat{L}_{n<0}$ onto $\{|\psi_{h_i}\rangle\}$, $\{|\psi_{h_0}\rangle\}$

- The bulk-edge correspondence

Ground state with Ne



↳ a virtual edge = a physical edge

$$\rho_A = \sum_i \lambda_i |\psi_{iA}\rangle\langle\psi_{iA}|$$

this basis spans the same conformal Hilbert space at $Ne/2$ electrons in the limit of $Ne \to \infty$

- The Hilbert space of a single LL is a conformal Hilbert space

  - It is the null space of the kinetic Hamiltonian

  - It is the space spanned by states satisfying $\hat{C} = [1, 1, 1]$

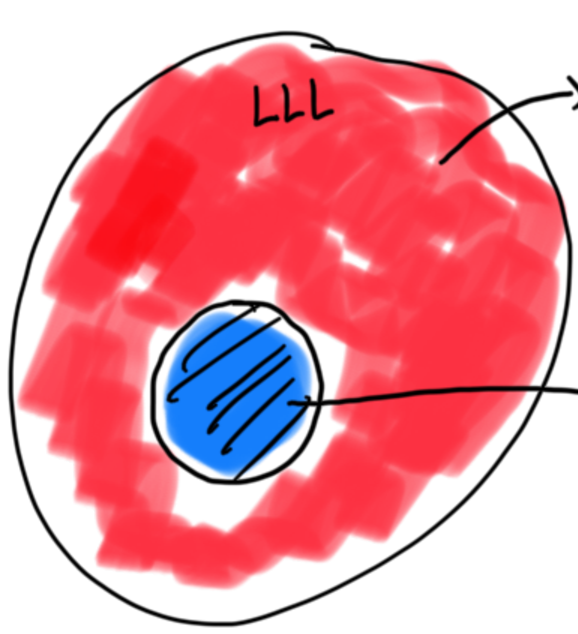
- Other examples of <u>Conformal Hilbert spaces</u>

  - Null space of model Hamiltonian ✓

  - Space spanned by a particular family of Jack polynomials satisfying certain admissible rules ✓

  - Space spanned by states satisfying <u>LEC</u>

- Hierarchy of CHS:



"quasiparticles"

<u>"anyons"</u> → elementary particles carrying a charge $\frac{e}{3}$ and a spin of $\frac{1}{6}$

 → null space of V_1

quarks

E

electrons

$\downarrow H_{kinetic}$

quasiparticles

$\downarrow V_1$

Laughlin-$\frac{1}{3}$ anyons — $H = V_1$, $\hat{C} = [2,1,2]$

$\downarrow V_3$

Laughlin-$\frac{1}{5}$ anyons — $H = V_1 + V_3$, $\hat{C} = [3,1,3]$

$\downarrow$

Laughlin $\frac{1}{7}$ anyons

$\simeq$

electrons, protons, neutrons

$\downarrow$ ionization and nuclear energy

H and O atoms

$\downarrow$ molecular bond

water molecules

$\downarrow$ Van der Waal

density waves

$\downarrow$

- Let $\mathcal{H}_{\frac{1}{2m+1}}$ be the null space of Laughlin-$\frac{1}{2m+1}$

$$\mathcal{H}_{\frac{1}{2m+1}} \subset \mathcal{H}_{\frac{1}{2m'+1}} \quad \text{for } m > m'$$

- $\mathcal{H}_{\frac{1}{3}} \subset \mathcal{H}_{\text{Moore-Read}}$

  $\hat{C} = [2,1,2]$ $\hat{C} = [3,2,3]$

- The Read-Rezayi Series

$$\mathcal{H}_{k+1} \supset \mathcal{H}_k$$

$$\hat{C} = [k+1, k, k+1]$$

$\mathcal{H}_{k=1} \to$ Laughlin $\frac{1}{3}$

$\mathcal{H}_{k=2} \to$ Moore-Read $\frac{2}{4}$

$\mathcal{H}_{k=3} \to$ Fibonacci - $\frac{3}{5}$

- The conformal Hilbert space formalism is a unifying picture for both IQH / FQH

  - Each conformal Hilbert space has a vacuum => the ground state of FQHE

  - Insertion of magnetic fluxes creates elementary particles -> <u>anyons</u>

  - All physics of FQH / IQH are determined by the algebraic properties of CHS.

  - Such physics is realised when the Hamiltonian creates an energy gap $\gg k_B T$ or disorder between $|\psi\rangle \in \mathcal{H}_{CHS}$ and $|\psi\rangle \notin \mathcal{H}_{CHS}$