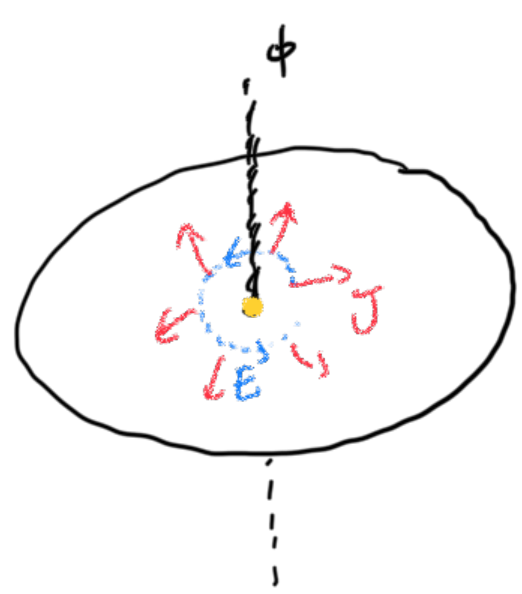
Lecture 6: - Properties of Anyons

- Anyons $\leftrightarrow$ quasiholes
  - created by insertion of magnetic fluxes into the ground state
    - $\hookrightarrow$ the vacuum of a CHS
  - Low lying excitations of the FQH phases (gapless) (topologically, they have zero energies)

> - A topological field theory has $H \sim p \cdot \dot{q} - L = 0$
> - All energy scales are sent to zero or infinity

  - Their properties are determined by the truncated Hilbert space they live in.

- The fundamental object is the magnetic flux.
  - The charge of the anyon $\underset{=}{e^*} = e \cdot \underset{=}{\nu}$ $\hookrightarrow$ Hall conductivity

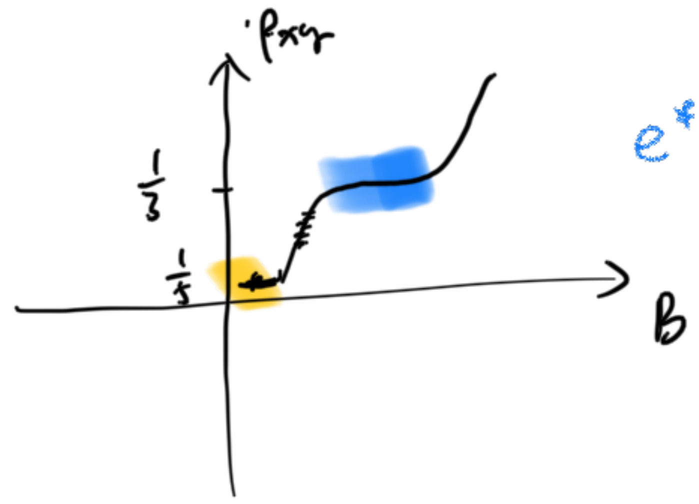


- the charge deficiency after an insertion of a single magnetic flux is $e \cdot \nu$

  - For example at $\nu = \frac{1}{3}$, an insertion of a magnetic flux gives $e^* = \frac{e}{3}$

- For systems without disorder,
  e^* depends on ν continuously
  (not topological)

- With disorder, ν forms a plateau



e^* is robust on the plateau

- Starting from Laughlin-$\frac{1}{3}$ ground state

Laughlin-$\frac{1}{3}$ of electrons $\rightarrow$ $N_0 = 3N_e - 2$ (vacuum)

After insertion of $\underline{2N_e - 2}$ fluxes

$\rightarrow \underline{N_0} = \underline{5N_e} - \underline{4}$

The $2N_e - 2$ anyons form a quantum fluid

With $H = V_1$, anyons are non-interacting
(ideal free gas of anyons)

With $H = V^{2body} = \sum_m C_m V_m$ $\hookrightarrow$ pseudo-potentials

$= C_1 V_1 + \sum_{m \geq 3} C_m V_m$

$\downarrow$
leads to interaction between anyons

for $\underline{\sum_{m \geq 3} C_m V_m \sim V_3}$, the $2N_e - 2$ $\underline{\underline{\text{anyons}}}$

form its own incompressible quantum fluid

$\leftrightarrow$ Laughlin-$\frac{1}{5}$ state of electrons

(Now each Laughlin-$\frac{1}{3}$ anyon carries a charge of $e^* = \frac{e}{5}$)

$\boxed{\text{※. Anyon charge is not "fundamental". The flux quantum is fundamental}}$

. Stacking of magnetic fluxes

- Starting with the ground state wavefunction in the LLL

$$\psi_0(z_1, z_2, \cdots z_{N_e})$$

. An insertion of q magnetic fluxes at position α is given by

$$\psi_\alpha^q(z_1, \cdots z_{N_e}) = \prod_i (z_i - \alpha)^q \, \psi_0(z_1, \cdots z_{N_e})$$

- This is a q-stacked quasihole with charge $q \cdot e^* = q e \nu$

- We can create two stacked quasiholes

$$\psi_{\alpha_1, \alpha_2}^{p,q} = \prod_i (z_i - \alpha_1)^p \cdot \prod_j (z_j - \alpha_2)^q \cdot \psi_0$$

An adiabatic braiding will give a Berry phase

$$\gamma = \gamma_B + \gamma_b$$

$$\gamma_B = \pi R^2 \cdot B \cdot q \cdot eV$$

$$\gamma_b = 2\pi \cdot p \cdot q \cdot \nu \quad \text{for large } R$$

magnetic flux

- Proof. Starting from the ground state ($\to L_z = 0$) on the sphere

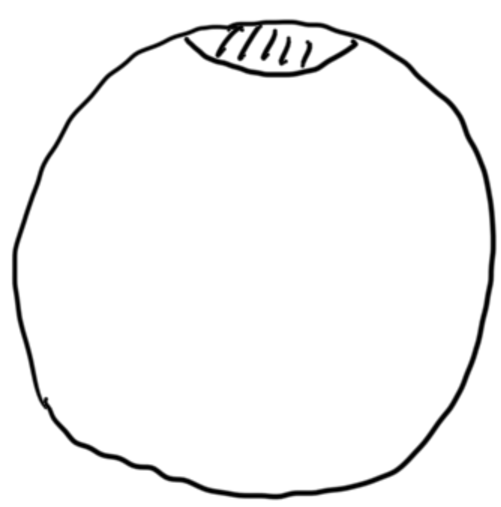
$$N_\phi + 1 = N_0 = \frac{p}{q}(N_e + S_e) - S_h$$

$$= \nu^{-1} \cdot N_e - S \qquad S = \frac{p}{q} S_e - S_h$$

$N_\phi = 2Q \to$ strength of the monopole at the center of the sphere

total magnetic flux

a. (p+q)-quasihole



$$L_{z,1} = -\frac{1}{2} N_e \cdot (p+q)$$

A 2π-rotation about z-axis gives

$$\gamma_1 = -\frac{1}{2} N_e (p+q) \cdot 2\pi$$

b. p-quasihole

q-quasihole

$$L_{z,2} = -\frac{1}{2} N_e (p-q)$$

A 2π-rotation:

$$\gamma_2 = -\frac{1}{2} N_e (p-q) \cdot 2\pi$$

$$\Delta\gamma_{p+q} = \gamma_2 - \gamma_1 = -\frac{1}{2} N_e \cdot 2\pi \cdot (-2q)$$

$$= -\frac{1}{2} \cdot \nu \cdot (-2q) \cdot (N_\phi + 1 + S - p - q) \cdot 2\pi$$

$$L_z = \vec{r}\times(\vec{p}-eA) = a^\dagger a - b^\dagger b$$

$$= v\cdot q\cdot N_\phi\cdot 2\pi$$

--> Berry phase from magnetic field

$$+ \frac{v\cdot q}{2}\cdot(1+2n+s)\cdot 4\pi$$

cyclotron angular momentum (n is the LL index)

guiding center angular momentum

↳ Berry phase from the coupling between the total angular momentum of the q-quasihole to the curvature of the sphere

$$-\frac{v\cdot q}{2}\cdot(p+q)\cdot 4\pi$$

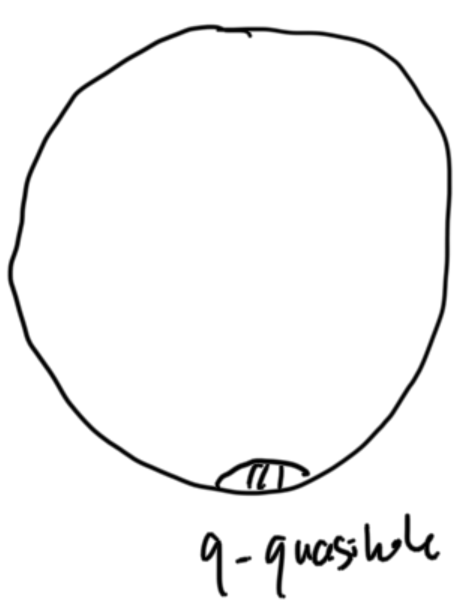
c.

$$L_{z,3} = -\frac{1}{2}N_e\cdot q$$

A 2π-rotation:

$$\gamma_3 = -\frac{v q}{2}(N_\phi+1+s-q)\cdot 2\pi$$

d.



$$L_{z,4} = \frac{1}{2}N_e\cdot q.$$

A 2π-rotation:

$$\gamma_4 = \frac{v q}{2}(N_\phi + \underset{+2n}{1+s} - q)\cdot 2\pi$$

$$\Delta\gamma_q = \gamma_4-\gamma_3 = 2\pi\cdot v\cdot q\cdot N_\phi$$

--> magnetic field

$$+ 4\pi\cdot\frac{v\cdot q}{2}(\underset{+2n}{1+s}-q)$$

↳ coupling of angular momentum to the curvature

$$\Rightarrow \quad \Delta \gamma = \underline{\Delta \gamma_{p+q}} - \underline{\Delta \gamma_q} = 4\pi \cdot \frac{\nu \cdot q}{2} \cdot (-p)$$
$$= -2\pi\nu \cdot p \cdot q$$

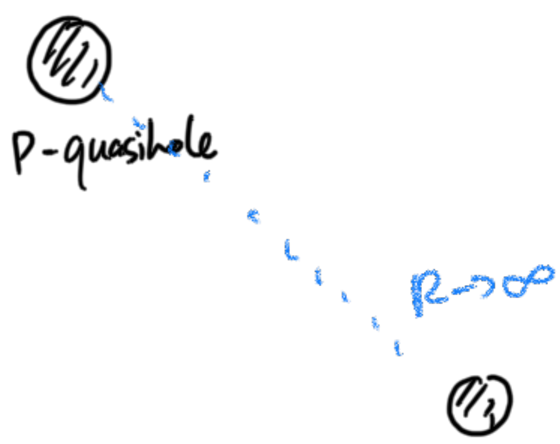
- A conformal mapping to the disk.

a

p+q-quasihole

$\underline{\tilde{\gamma}_1}$ (self-rotation of p+q-quasihole)

b.



$\underline{\tilde{\gamma}_2}$ (self-rotation of p-quasihole and q-quasihole, and braiding)

$$\Delta \tilde{\gamma}_{p+q} = \tilde{\gamma}_2 - \tilde{\gamma}_1 = \Delta \gamma_{p+q} - \underline{\text{curvature contribution}}$$

c.

q-quasihole

$\tilde{\gamma}_3$ (self-rotation of q-quasihole)

d

$\tilde{\gamma}_4$ (self-rotation + the loop)

$$\Delta \tilde{\gamma}_q = \tilde{\gamma}_4 - \tilde{\gamma}_3 = \Delta \gamma_q - \text{curvature contribution}$$

Loop Containing p-quasihole

Loop without a p-quasihole

$$\Rightarrow \quad \Delta\tilde{\gamma}_b = \Delta\tilde{\gamma}_{p+q} - \Delta\tilde{\gamma}_q = \Delta\gamma = -2\pi\cdot\nu\cdot p\cdot q$$

$$= 2\pi\cdot\left(\frac{\nu}{2}(p+q)^2 - \frac{\nu}{2}p^2 - \frac{\nu}{2}q^2\right)$$

Braiding phase

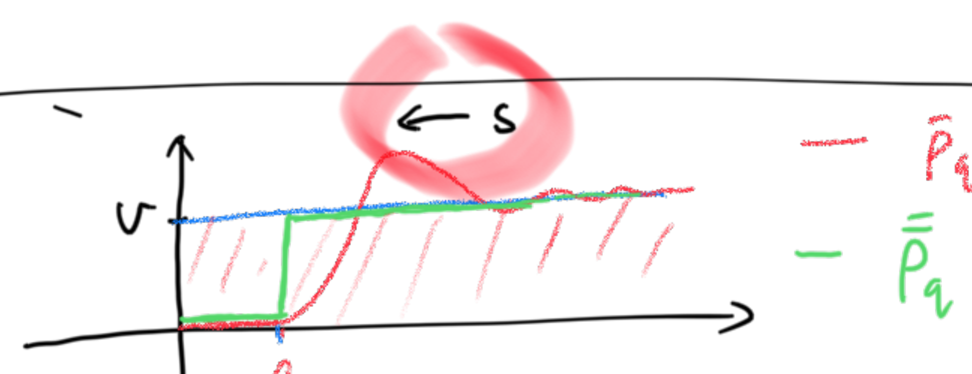
topological spin of (p+q)-quasihole

... of p-quasihole

... of q-quasihole

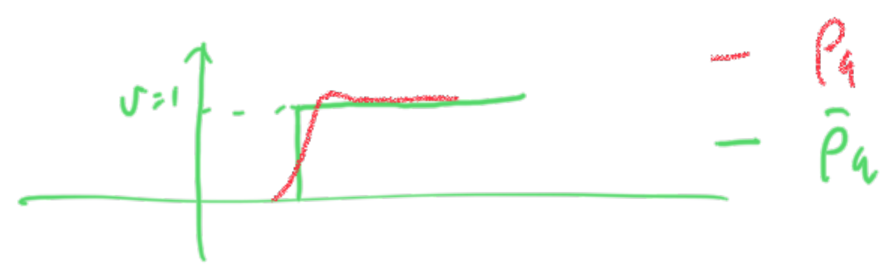
$\Rightarrow$ For a k-quasihole, its topological spin is $\frac{\nu}{2}\cdot k^2$, that satisfies the spin-statistics theorem

a self rotation of the particle by 2π gives a phase of $2\pi\cdot\frac{\nu}{2}\cdot k^2$



$$\Delta L_z = \Delta L_z^{q\text{-quasihole}} - \Delta L_z^{\text{groundstate}} = -\frac{1}{2}\nu\cdot q^2 + \frac{1}{2}\nu\cdot s$$

$$\frac{1}{2}\nu\cdot s = \sum_{m\geq q}(n_m-\nu)\cdot\left(m+\frac{1}{2}\right)$$

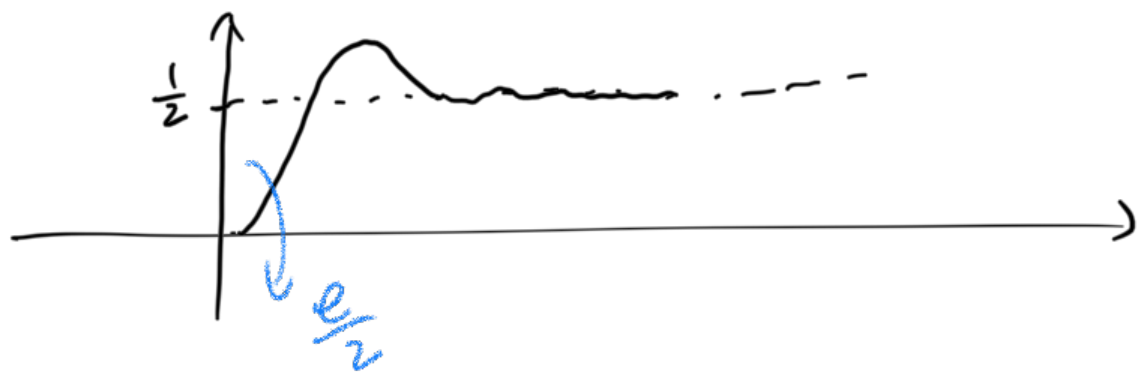


· Fractionalization of magnetic fluxes

· A single magnetic flux insertion to the Moore-Read state.

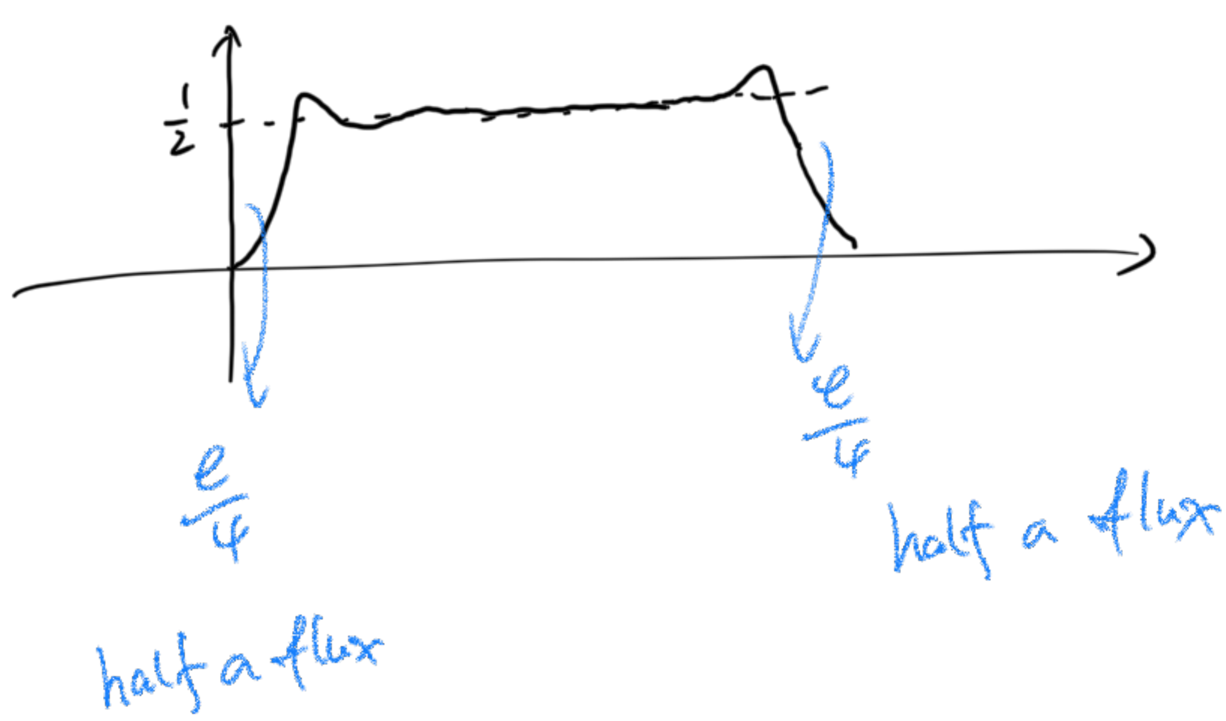
0110011001100110011

this is a "quasihole" of charge $\frac{1}{2} \cdot e$



· But there are other zero energy states (or states in the MR-conformal Hilbert space) with one magnetic flux.

1010101010101010101



· Splitting / fractionalization of a magnetic flux

$\cdot \to \frac{e}{4}$

0110011001100110011

1010011 0011001100 11

1010101 0011 0011 0011

10101010101 0011 0011

... 10101 0011

|0|0|0|0|0|0|0|0| . . .

|0|0|0|0|0|0|0| 0|0|0|0|0|.

- Thus, each magnetic flux creates two quasiholes, each with charge $\frac{e}{4}$
  (non-Abelions)

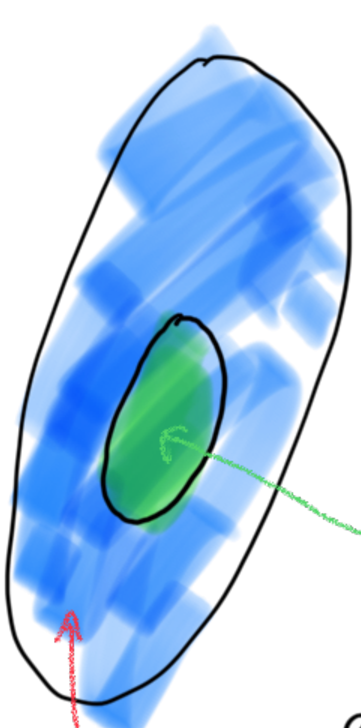
- The same holds for Gaffnians $\left(e^* = \frac{1}{5}e,\ \nu = \frac{2}{5}\right)$
  and Haffnians $\left(e^* = \frac{1}{6}e,\ \nu = \frac{2}{6}\right)$

- For the Fibonacci state at $\nu = \frac{3}{5}$,
  each magnetic flux creates 3 quasiholes $\left(e^* = \frac{1}{5}\right)$

- $\mathcal{H}_{\nu=\frac{1}{3}} \subset \mathcal{H}_{\nu=\frac{2}{5}}$

  $\mathcal{H}_{\nu=\frac{1}{3}}$ ↺ CHS of Laughlin-$\frac{1}{3}$

  $\mathcal{H}_{\nu=\frac{2}{5}}$ ↺ CHS of Gaffnian at $\nu = \frac{2}{5}$



11000 1100011 → Gaffnian ground (vacuum)

↓ insert 4 fluxes

Laughlin ground state as a quantum fluid of Gaffnian quasiholes ← |00|00 |00|00|00|

• → Gaffnian quasiholes

• → Laughlin quasihole

↓

a bound state of two Gaffnian quasiholes

↓ insert 1 more flux

0|00 |00 |00 |00|00|

↓ → Laughlin-neutral excitations (gluons)

Laughlin

0 | 0 | 000 | 0 | 0000 | 0 | 0

0 -> Laughlin quasielectron (charged excitation)

A single flux -> a single Laughlin-$\frac{1}{3}$ quasihole

-> a bound state of two Gaffnian quasiholes

each with half a flux

for short range interaction, bound states are energetically favourable

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