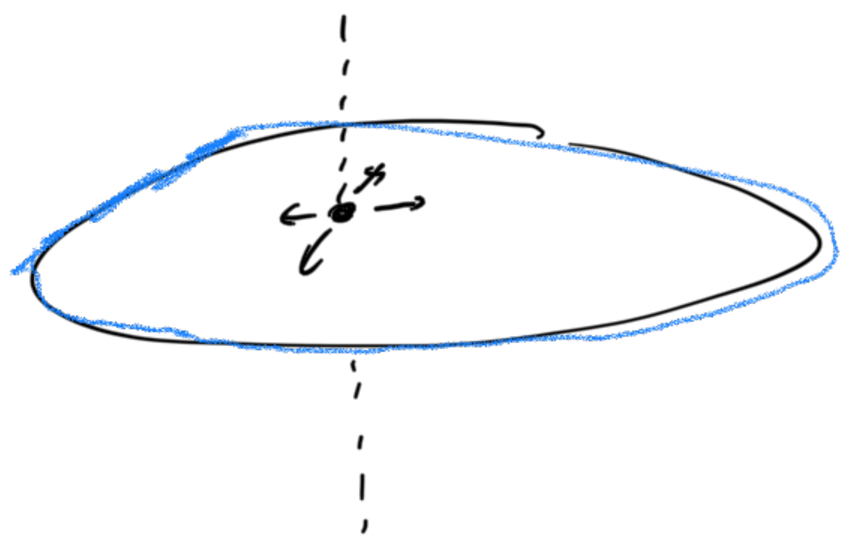
• Lecture 7: <u>Bosonization in 2D</u>

(PRL 127, 126406)

– The bulk-edge correspondence



• An insertion of magnetic flux creates an electron deficiency (quasiholes)

• Radial current pushes electrons to the edge

• The dual description.

quasihole in the bulk

$\longleftrightarrow$ density modulation at the edge

– Using the Laughlin state as an example

$(z_i - z_j)^m \quad m=3, \quad M = \frac{3}{2} N_e (N_e - 1)$

ground state ... 100 100 100 1001 $\quad \Delta M = 0$

The quasihole states.

$\Delta M = 1$

$1 = 1$ ... 100 100 100 10001

$2 = 1+1$ ... 100 100 100 100001

$2 = 2$ ... 100 100 1000 1001

$\Delta M = 2$

$3 = 1+1+1$ ... 100 100 100 1000001

$3 = 2+1$ ... 100 100 1000 10001

... 100 100 0100 1001

$\Delta M = 3$

$3=3$

⋮

- The number of quasihole states in each ΔM sector is given by $P(\Delta M)$ → Virasoro Counting of the chiral Luttinger liquid

¿ CFT

integer partition

- Assuming the energy spectrum of quasihole states as

$$E_{\Delta M} = V_F \Delta M \qquad \Delta M = R \cdot \Delta k$$

↳ linear momentum along the edge

The partition function of quasiholes (1D edge system)

$$Z = \sum_{\Delta M=0}^{\infty} P(\Delta M) e^{-\beta V_F \Delta M}$$

The heat capacity

$$C = \frac{\partial E}{\partial T} = \frac{\partial}{\partial T}\left(\frac{\partial}{\partial \beta}\log Z\right) \sim 1$$

↓ Thermal Hall coefficient (central charge of the CFT edge)

↓ universal in the limit $\beta/R \ll 1$, independent of V_F

- $P(\Delta M)$ → density of states
$E_{\Delta M} = V_F \Delta M$ → linear dispersion

} Conformal symmetry of the 1+1 edge dynamics
⇒ the entire spectrum

can be bosonized

- For finite systems with N_e electrons, the quasihole counting is given by $P(\Delta M, N_e) \le P(\Delta M, \infty) = P(\Delta M)$

integer partition of ΔM into non-negative integers no larger than N_e

(Finite systems break conformal symmetry)

For $\Delta M \le N_e$, $P(\Delta M, N_e) = P(\Delta M)$

$\Delta M > N_e$ $\quad P(\Delta M, N_e) < P(\Delta M)$

$\Delta k = \frac{\Delta M}{R}$, conformal symmetry is only broken at large Δk

- Conformal mapping from disk to sphere.

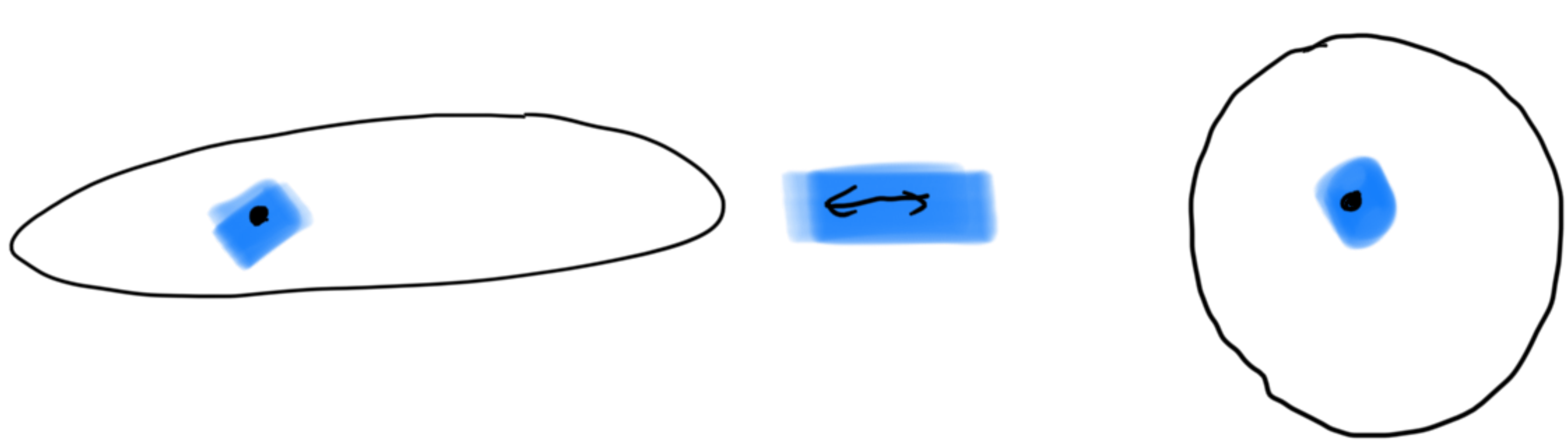
. The stereographic mapping

$$z^n e^{-\frac{1}{4}zz^*} \longleftrightarrow z^n \left(\frac{1}{1+zz^*}\right)^s$$

2s is total magnetic flux

on the disk $z = x+iy$

on the sphere $z = \tan\theta/2 \, e^{i\varphi}$



. A quasihole on the sphere has no dual interpretation: if is a bulk anyonic excitation in 2D

- Quantum numbers of magnetic fluxes on the sphere.

  - The ground state of FQHE with N_e electrons is in $L_z = 0$ sector ($L_z = 0$, $L^2 = 0$)

    => (ground state is a rotationally invariant quantum fluid)

$$\boxed{N_\phi = \frac{p}{q}(N_e + S_e) - S_h}$$

- Using the IQHE as an example.

  - A single magnetic flux has a quantum number of a single particle on the sphere

    $|m, l\rangle$ ($L_z|m,l\rangle = m|m,l\rangle$, $L^2|m,l\rangle = l(l+1)|m,l\rangle$)

    $l = \frac{1}{2}N_e$ => a spinor of $S = \frac{1}{2}N_e$

    (we do not allow fractionalization of magnetic fluxes

    => abelian Conformal Hilbert space)

    $0 1 1 1 1 \cdots 1 \to |l, l\rangle$

    $1 0 1 1 1 1 \cdots 1 \to |l-1, l\rangle = L^+ |l, l\rangle$

    $\vdots$

- Insertion of k magnetic fluxes

  $|m_1 m_2 \cdots m_k, l\rangle$ $\quad l = \frac{N_e}{2} + \frac{1}{2}(k-1)$

each magnetic flux is a spinor of $S = \frac{1}{2}(Ne + k - 1)$

fermionic (anyonic)

Total no. of States $= \dfrac{(Ne+k)!}{k!\, Ne!}$

- Diagonalization of this k-flux Hilbert space with L^2: $[L_z, L^2] = 0$

$|M, L\rangle \longrightarrow$ eigenstates come from k bosonic particles, each a spinor with $S_b = \frac{Ne}{2}$

M: total L_z, L: total L^2

independent of k

- For $k=2$, total number of states is $\frac{1}{2}(Ne+2)(Ne+1)$

$L = Ne,\ Ne-2,\ Ne-4, \ldots$

$M: -L, -L+1, \ldots\ L-1, L$

· For $k=3$, the allowed L value is

$$\underbrace{L = \tfrac{3}{2}Ne,\ \tfrac{3}{2}Ne-2,\ \tfrac{3}{2}Ne-3,\ \tfrac{3}{2}Ne-4,\ \tfrac{3}{2}Ne-5,\ \underbrace{\tfrac{3}{2}Ne-6}_{\text{two states}}^{*} \ldots}_{\text{Bosonic counting of three bosons}}$$

- Thus, each magnetic flux can either be treated

... of $S = \frac{1}{2}(Ne+k-1)$

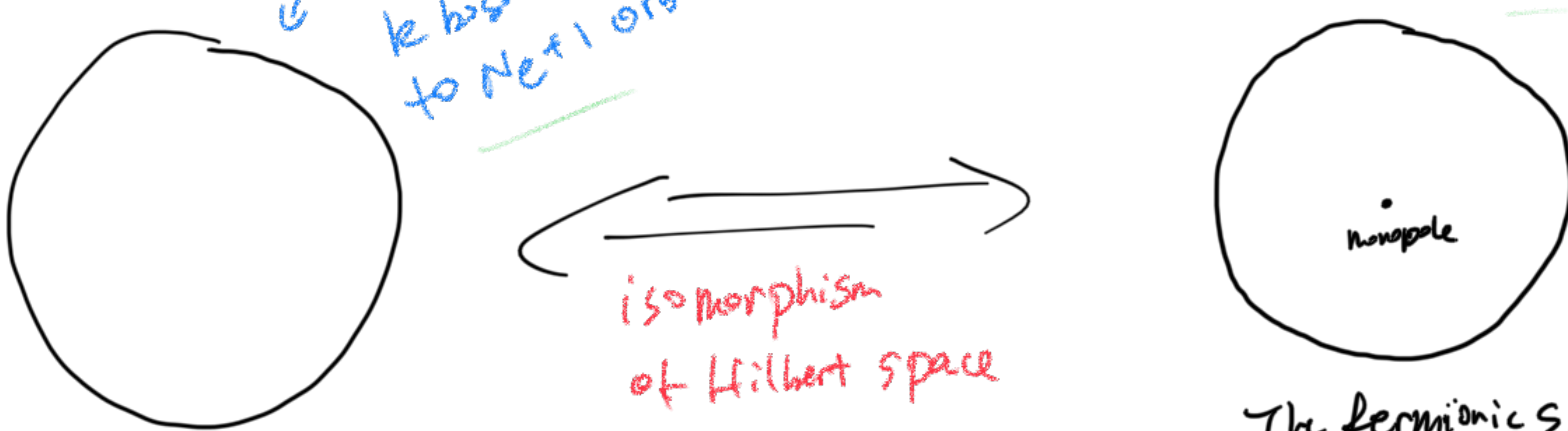
as a fermion (anyon) of

or a boson of $S=\frac{1}{2}N_e$

. The bosonization scheme.

adding k bosons to N_e+1 orbitals

N_e electron in N_e+k orbitals



isomorphism of Hilbert space

The bosonic sphere

$\underline{N_\phi' = N_e}$

The fermionic sphere

$N_\phi = 2Q = \underline{N_e + k - 1}$

$$|m_1', m_2', \dots m_k', \ell'\rangle = \sum_{\vec{m}} C^{\vec{m}', \ell'}_{\vec{m}, \ell} \, |m_1, m_2, \dots m_k, \ell\rangle$$

$\ell' = \frac{1}{2}N_e$

$-\ell' \le m_i' \le \ell'$

$\ell = \frac{1}{2}(N_e+k-1)$

$-\ell \le m_i \le \ell$

$= \sum_{M,L} C^{\vec{m}',\ell'}_{M,L} \underline{|M,L\rangle_B}$

eigenstates of L^2

$= \sum_{M,L} \tilde{C}^{\vec{m},\ell}_{M,L} \underline{|M,L\rangle_F}$

eigenstates of L^2

$$|M,L\rangle_F = \sum \left(\tilde{C}^{\vec{m},\ell}_{M,L}\right)^{-1} |m_1, m_2, \dots m_k, \ell\rangle$$

$$|M,L\rangle_B = \sum \left(C^{\vec{m}',\ell'}_{M,L}\right)^{-1} |m_1', m_2', \dots m_k', \ell'\rangle$$

$$\Rightarrow \underline{|m_1', m_2' \dots m_k', \ell'\rangle} = \sum_{\substack{M,L \\ \vec{m},\ell}} C^{\vec{m}',\ell'}_{M,L} \left(\tilde{C}^{\vec{m},\ell}_{M,L}\right)^{T} \underline{|m_1, m_2, \dots m_k, \ell\rangle}$$

a bosonic product state

a fermionic (anyonic) many body state

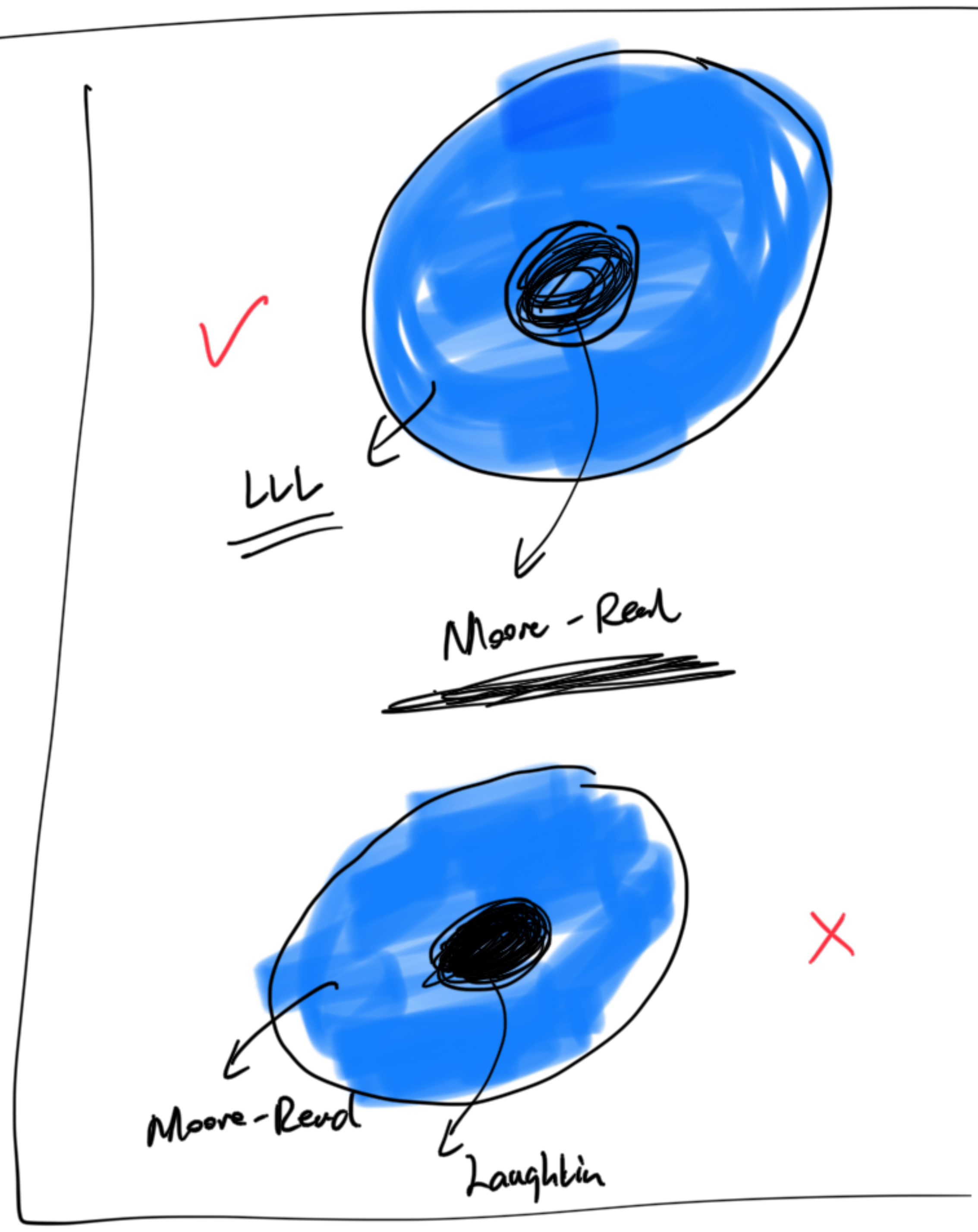
$C^{n/l}_{\bar{m}/l}$

$3000\cdots0\cdots \longleftrightarrow 000111111\cdots$

- The same procedure can be extended to FQH states, as long as magnetic flux does not fractionalize (e.g. Laughlin states)

- Bosons are linear combinations of anyons (a unitary transformation)

- Exact bosonization in 2D for anyons for any finite systems



- The statistical interaction

  - Let H_F be the interaction between electrons

$$ {}_B\langle M, L, k \mid H^B_{eff} \mid M, L, k\rangle_B = {}_F\langle M, L, k \mid H_F \mid M, L, k\rangle_F $$

H^B_{eff} ↓ on the bosonic sphere

H_F ↓ on the fermionic sphere

- Note that $H^B_{eff} = \sum_{k,M} C_{k,M} V^{k\text{-body}}_{L-M}$

we have

$$ C_{k,L-M} = {}_B\langle M, L, k | H^B_{eff} | M, L, k\rangle_B - \sum_{\substack{k'<k \\ M'}} C_{k',L-M'} \, {}_B\langle M, L, k | V^{k'\text{-body}}_{L-M'} | M, L, k\rangle_B $$

$$ = {}_F\langle M, L, k | H_F | M, L, k\rangle_F - \sum_{\substack{k'<k \\ M'}} C_{k',L-M'} \, {}_B\langle M, L, k | V^{k'\text{-body}}_{L-M'} | M, L, k\rangle_B $$

$\Rightarrow$ $C_{k,m}$ is determined by H_F

→ interaction between electrons

- Example: For $H_F = V_1^{2bdy}$ → model Hamiltonian for Laughlin-$\frac{1}{3}$

$$\Rightarrow H_{eff}^{B} = V_0^{2bdy} - 0.737\, V_0^{3bdy} + 0.177\, V_2^{3bdy} + \cdots$$

↓ a model Hamiltonian for $\nu = 2$ bosons

↓ interaction between bosons that captures the statistical interaction between fermions/anyons

- Every FQH state has one or more bosonic dual partner.

	LLL CHS $1111\cdots111 \to$ vacuum	V_1-CHS $1001001001\cdots \to$ vacuum
Laughlin-$\frac{1}{3}$ state $N_\phi = 3N_e - 2$	Bosonic $\nu = 2$ $N_\phi = \frac{1}{2}N_e + 2$	
Moore-Read-$\frac{1}{2}$ state $N_\phi = 2N_e - 2$	Bosonic $\nu = 1$ $N_\phi = N_e + 3$	
Laughlin-$\frac{1}{5}$ state $N_\phi = 5N_e - 4$	Bosonic $\nu = 4$ $N_\phi = \frac{1}{4}N_e + 2$	Bosonic $\nu = 2$ $N_\phi = \frac{1}{2}N_e + 2$

$$C_{k',L-m'}\ {}_{B}\langle m, L, k \,|\, V^{k'\text{-body}}_{L-m'} \,|\, m, L, k\rangle_{B}$$