Universal entropy of conformal critical theories on a Klein bottle: a quantum Monte Carlo study

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Collaborators

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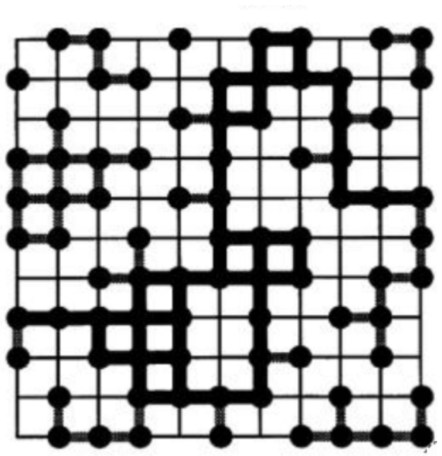
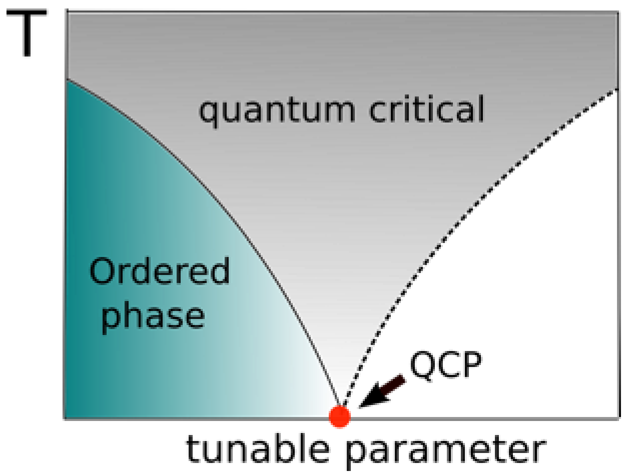
Critical phenomena and conformal field theory

- Conformal invariance at the critical points

- In 2D, the conformal invariance gives very strong constraints

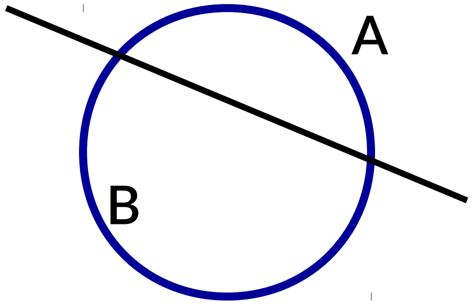
  → critical points of 2D classical models or 1D quantum models





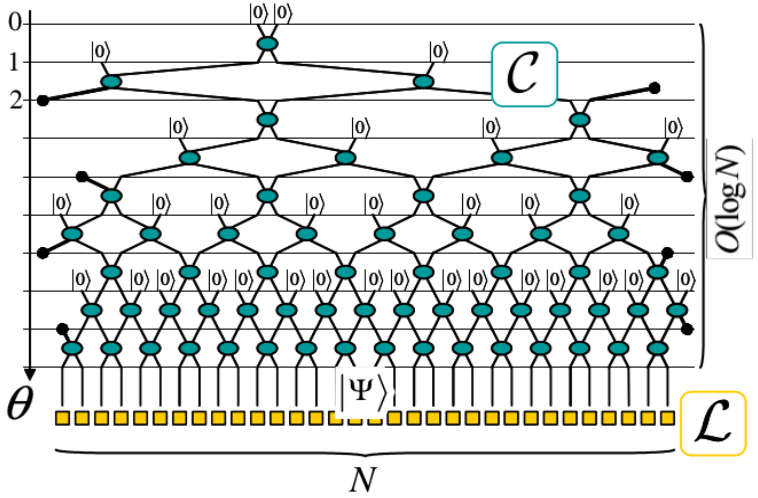
Address the underlying CFT of a microscopic system

- Entanglement entropy $S_A \sim \frac{c}{3} \ln L$



Calabrese and Cardy,
J. Stat. Mech. 2004,
J. Phys. A 2009

- MERA



Vidal, PRL 2008

This work → Klein bottle entropy: a universal quantity as a characterization of the CFT

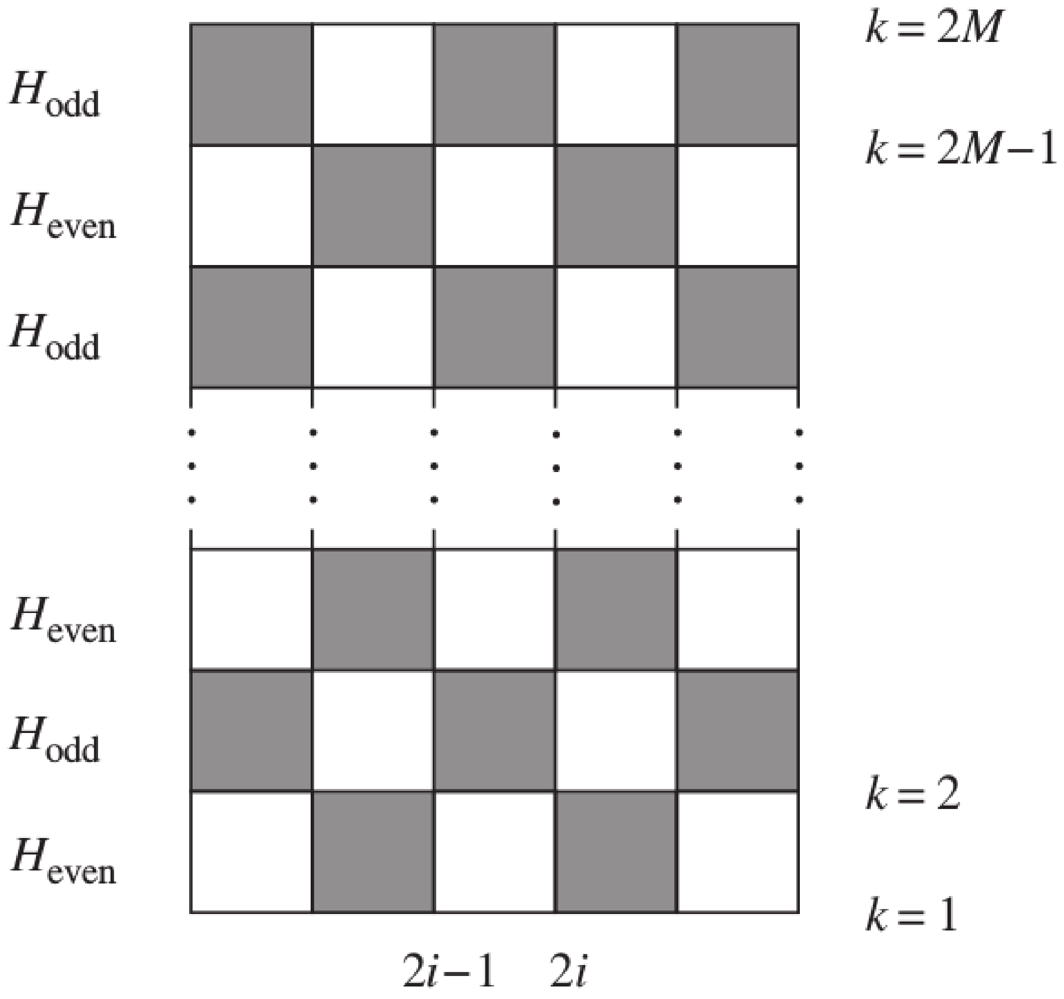
Outline

- Critical phenomena and conformal field theory
- Universal boundary entropies on various manifolds
- Calculations in lattice models
- Models and results
- Summary

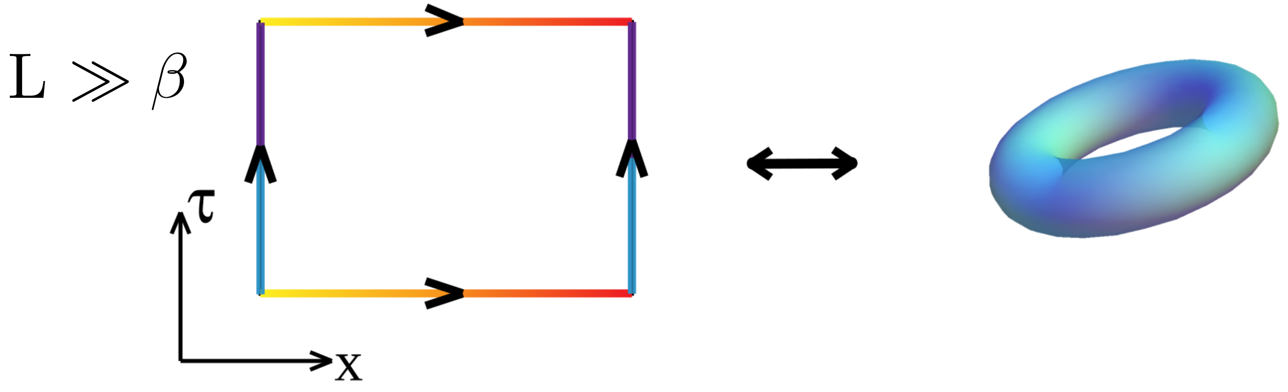
The 1D quantum system at finite temperature

$$
\begin{aligned}
Z^{\mathcal{T}} &= \mathrm{tr}(\mathrm{e}^{-\beta H}) \\
&= \mathrm{tr}(\mathrm{e}^{-\Delta\tau H_{\mathrm{even}}}\mathrm{e}^{-\Delta\tau H_{\mathrm{odd}}} \dots \mathrm{e}^{-\Delta\tau H_{\mathrm{odd}}})
\end{aligned}
$$

The partition function of a 1D quantum system with periodic boundary conditions lives on a torus



Torus partition function (of a long strip)

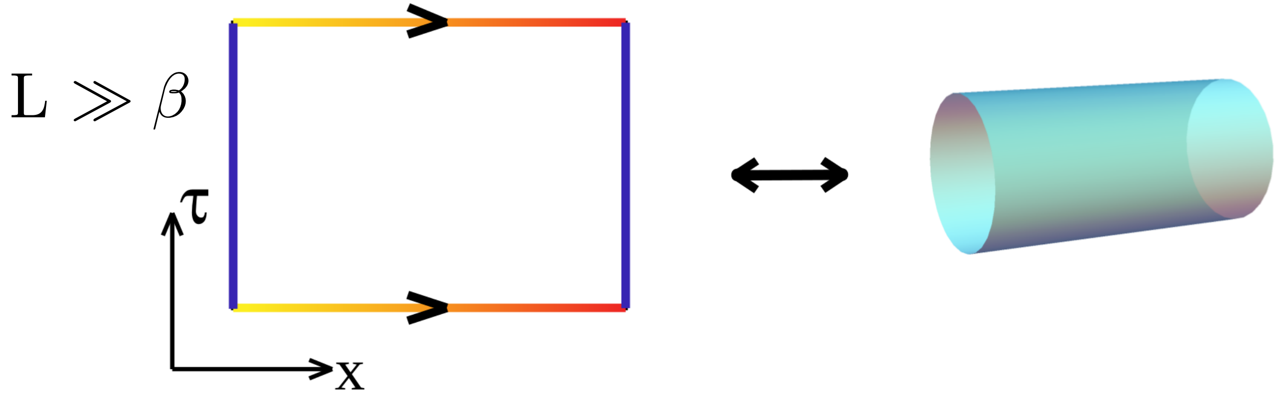


$$\ln Z^{\mathcal{T}} = -f_0 \beta L + \frac{\pi c}{6\beta v} L + O(\frac{1}{\beta^2})$$

- f_0 the bulk free energy density
- $\frac{\pi c}{6\beta c} L$ the bulk entropy (Prediction of CFT)

Bloete, Cardy and Nightingale, PRL 1986
Affleck, PRL 1986

Cylinder partition function (of a long strip)

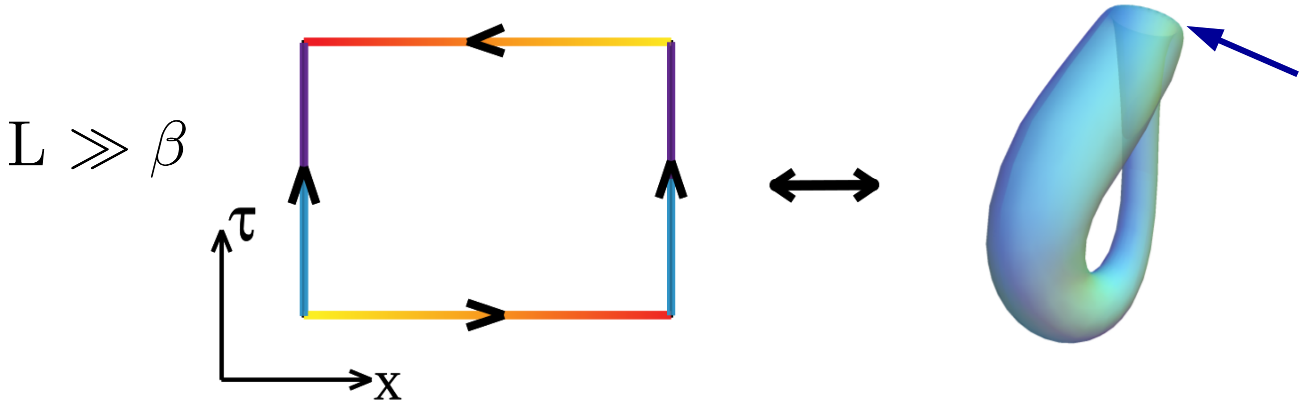


$$\ln Z^{\mathcal{C}} = -f_0 \beta L + \frac{\pi c}{6\beta v} L + S_{\mathrm{AL}} - f_{\mathrm{b}} \beta + O(\frac{1}{\beta^2})$$

- f_{b} the surface free energy density
- S_{AL} the universal Affleck-Ludwig entropy (Prediction of CFT)

Affleck and Ludwig, PRL 1991

Klein bottle partition function (of a long strip)



Spatial reflection operator Ω

Interchanges left- and right-movers

$$\ln Z^{\mathcal{K}} = -f_0 \beta L + \frac{\pi c}{24 \beta v} L + S_{\mathrm{KB}} + O(\frac{1}{\beta^2})$$

- Different coefficient of the bulk entropy
- An additional universal term S_{KB} resembling the AL entropy
  → The Klein bottle entropy

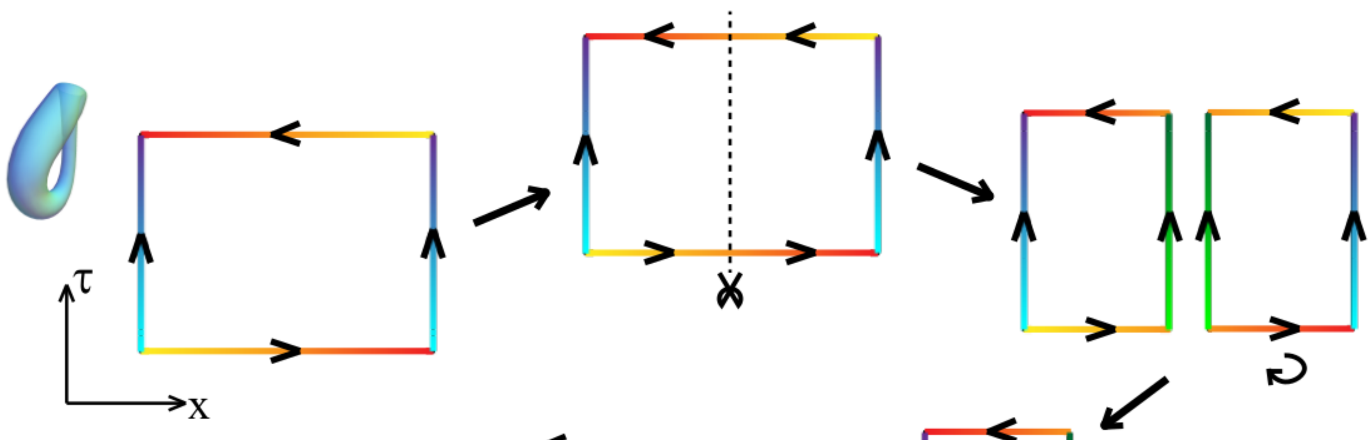
Tu, PRL 2017

Transformation of the Klein bottle

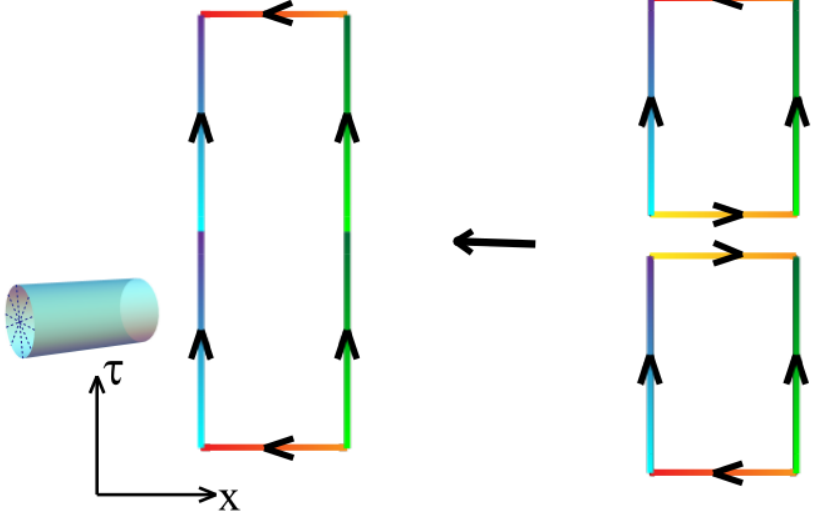
Klein bottle (L, β)

$$\frac{\pi c}{24\beta v} L = \frac{\pi c}{6(2\beta)v}\frac{L}{2}$$

cylinder $(L/2, 2\beta)$

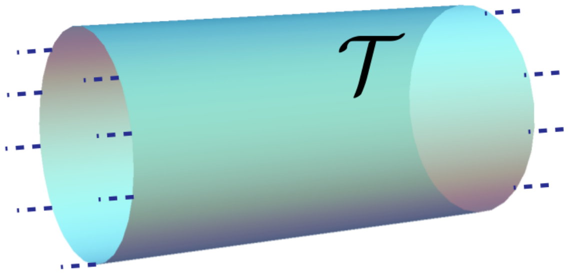


long-range interactions on
the boundary of the cylinder
→ "a boundary condition"



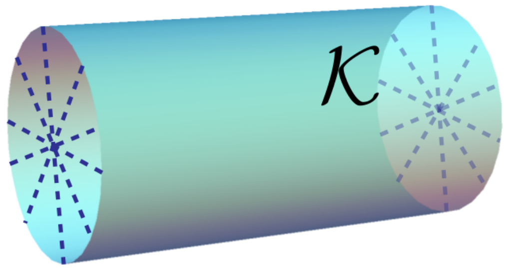
WT, Chen, Li, Xie, Tu and Wang, PRB 2017

Klein bottle entropy: a boundary entropy



$$\ln Z^{\mathcal{T}}(L,\beta) = -f_0 \beta L + \frac{\pi c}{6\beta v} L + O(\frac{1}{\beta^2})$$

emerges from the new boundary condition



$$\ln Z^{\mathcal{K}}(2L,\beta/2) = -f_0 \beta L + \frac{\pi c}{6\beta v} L + \boxed{S_{\mathrm{KB}}} + O(\frac{1}{\beta^2})$$

$$\Rightarrow \quad S_{\mathrm{KB}} = \ln\left[\frac{Z^{\mathcal{K}}(2L,\beta/2)}{Z^{\mathcal{T}}(L,\beta)}\right]$$

Extracts the ground-state property from thermal systems

CFT predictions of the Klein bottle entropy

- In rational CFT,

$$S_{\mathrm{KB}} = \ln\left(\sum_a M_{aa} d_a / \mathcal{D}\right)$$

d_a the quantum dimension of the primary field

Tu, PRL 2017

- In the compactified boson CFT,

$$S_{\mathrm{KB}} = \ln R$$

R the compactification radius

$\phi = \phi + 2m\pi R, m = 0, \pm 1, \ldots$

WT, Xie, Wang and Tu, 1805.01300

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Bond-centered lattice reflection

?

Lattice reflection

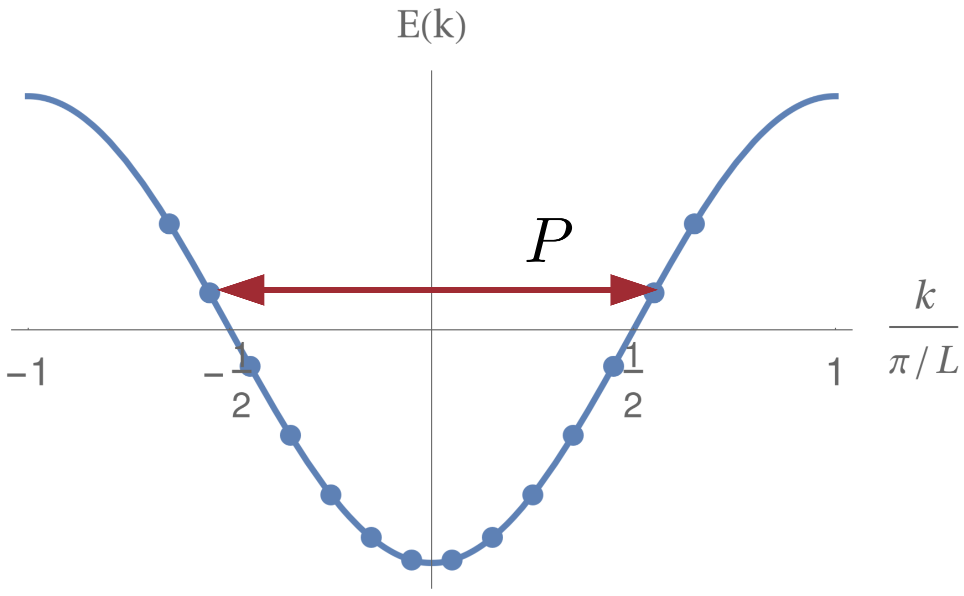
$$P|s_1,\ldots,s_L\rangle = |s_L,\ldots,s_1\rangle$$

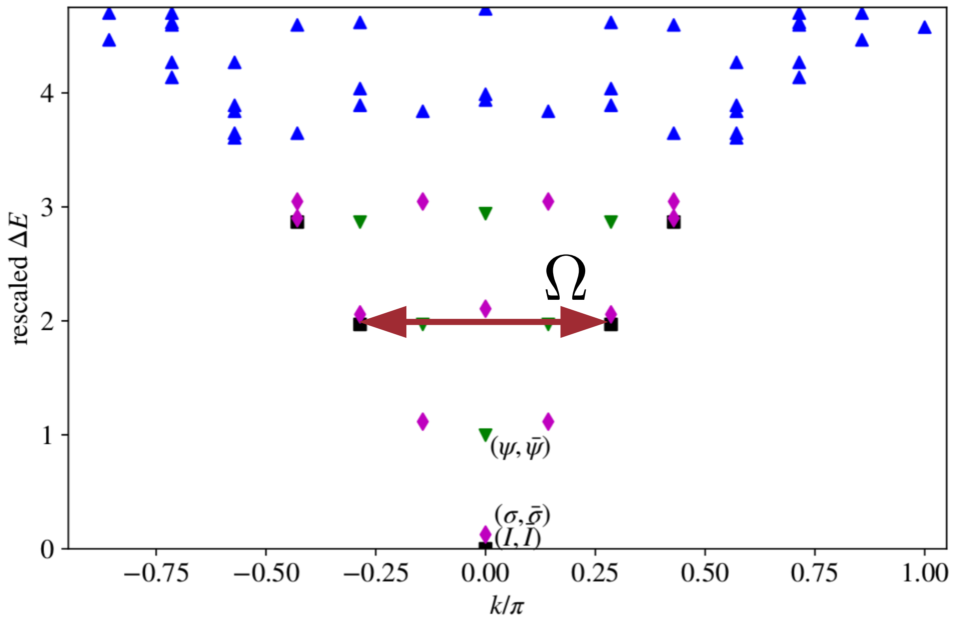
$$Z^{\mathcal{K}} = (P\mathrm{e}^{-\beta H})$$

The reflection Ω in the CFT Hilbert space

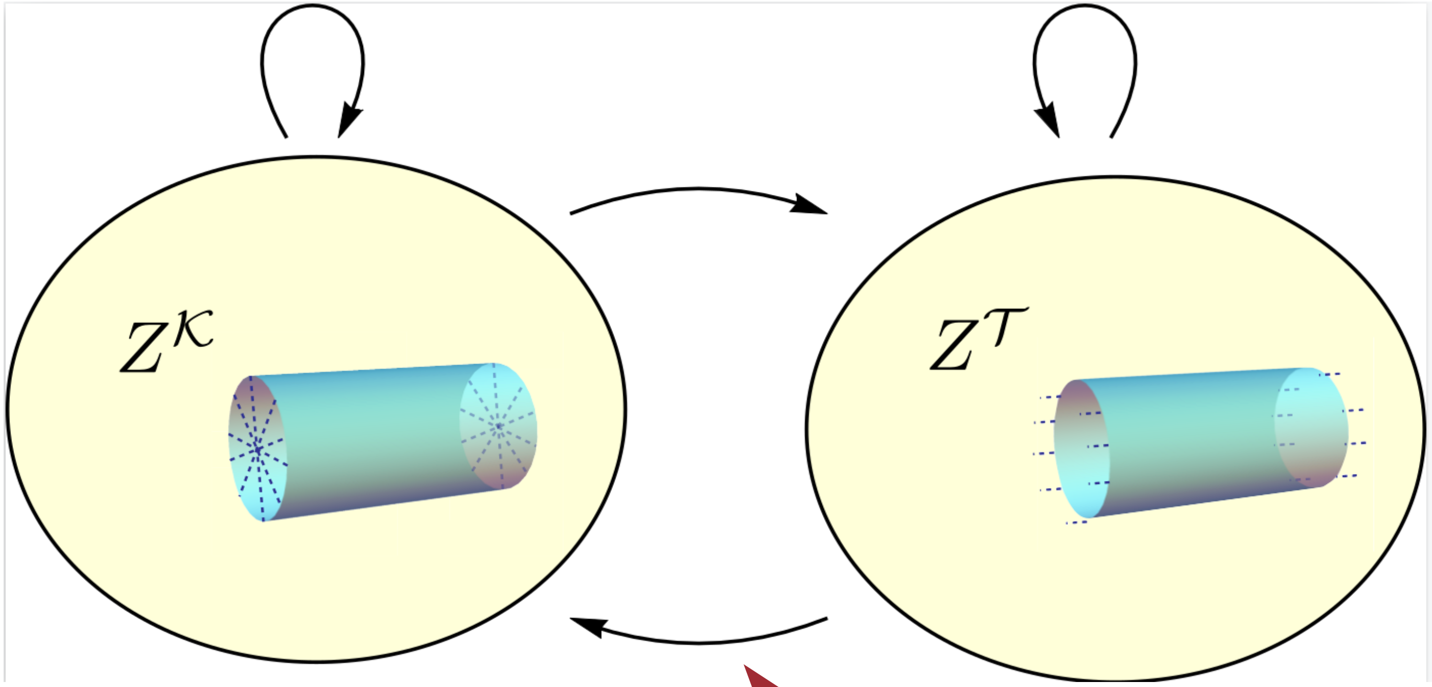
$$\Omega|\alpha,\bar{\gamma}\rangle = |\gamma,\bar{\alpha}\rangle$$

$$Z^{\mathcal{K}} = (\Omega\mathrm{e}^{-\beta H})$$





Extended-ensemble Monte Carlo for partition function ratios



$$g = \frac{Z^{\mathcal{K}}}{Z^{\mathcal{T}}}$$

Only involves manipulations on the boundaries

WT, Chen, Li, Xie, Tu and Wang, PRB 2017

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Spin-1/2 XY chain

$$H = -\sum_{i=1}^{L}(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y)$$

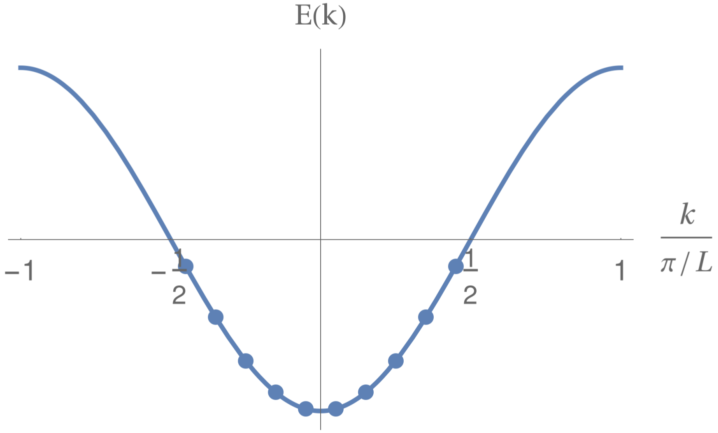
Jordan-Wigner

$$H_\pm = -\frac{1}{2}\sum_{i=1}^{L}(f_i^\dagger f_{i+1} + \text{h.c.})$$

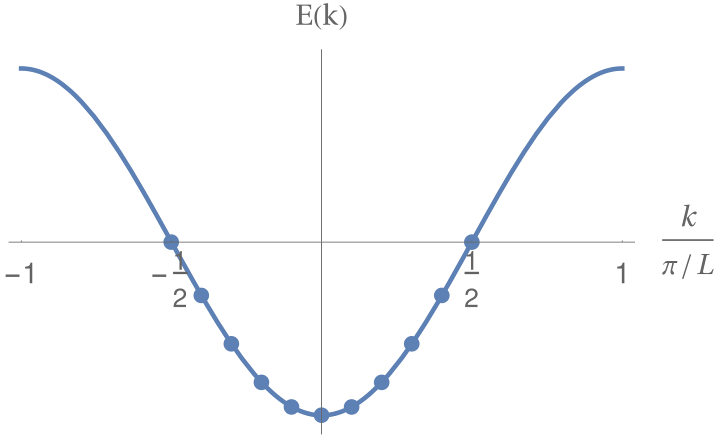
$$= -\sum_k \cos(k) f_k^\dagger f_k$$

NS sector: anti-PBC $f_1 = -f_{L+1}$ even number of fermions

R sector: PBC $f_1 = f_{L+1}$ even number of fermions

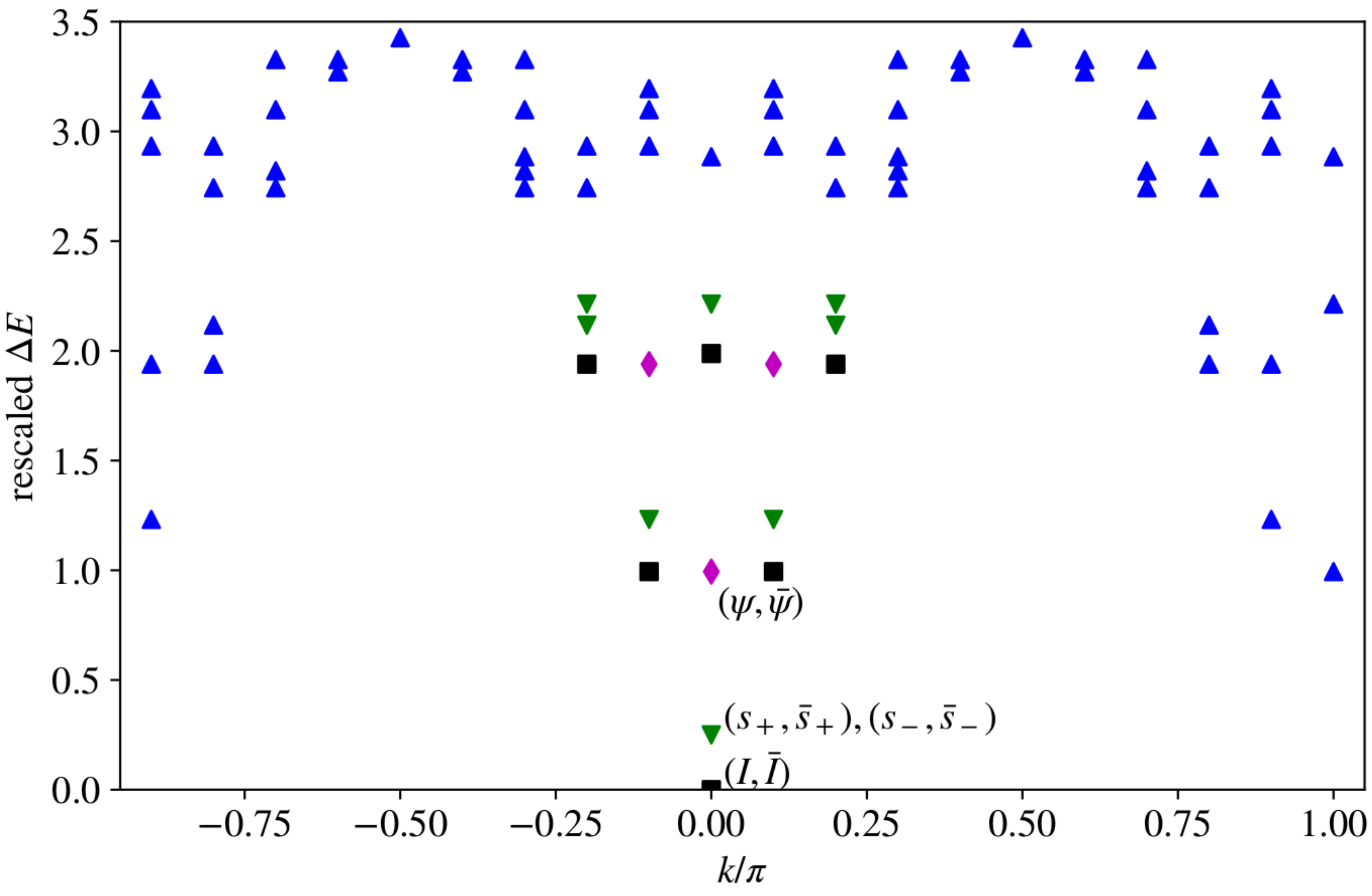


NS sector



R sector

Spin-1/2 XY chain (rational CFT analysis)



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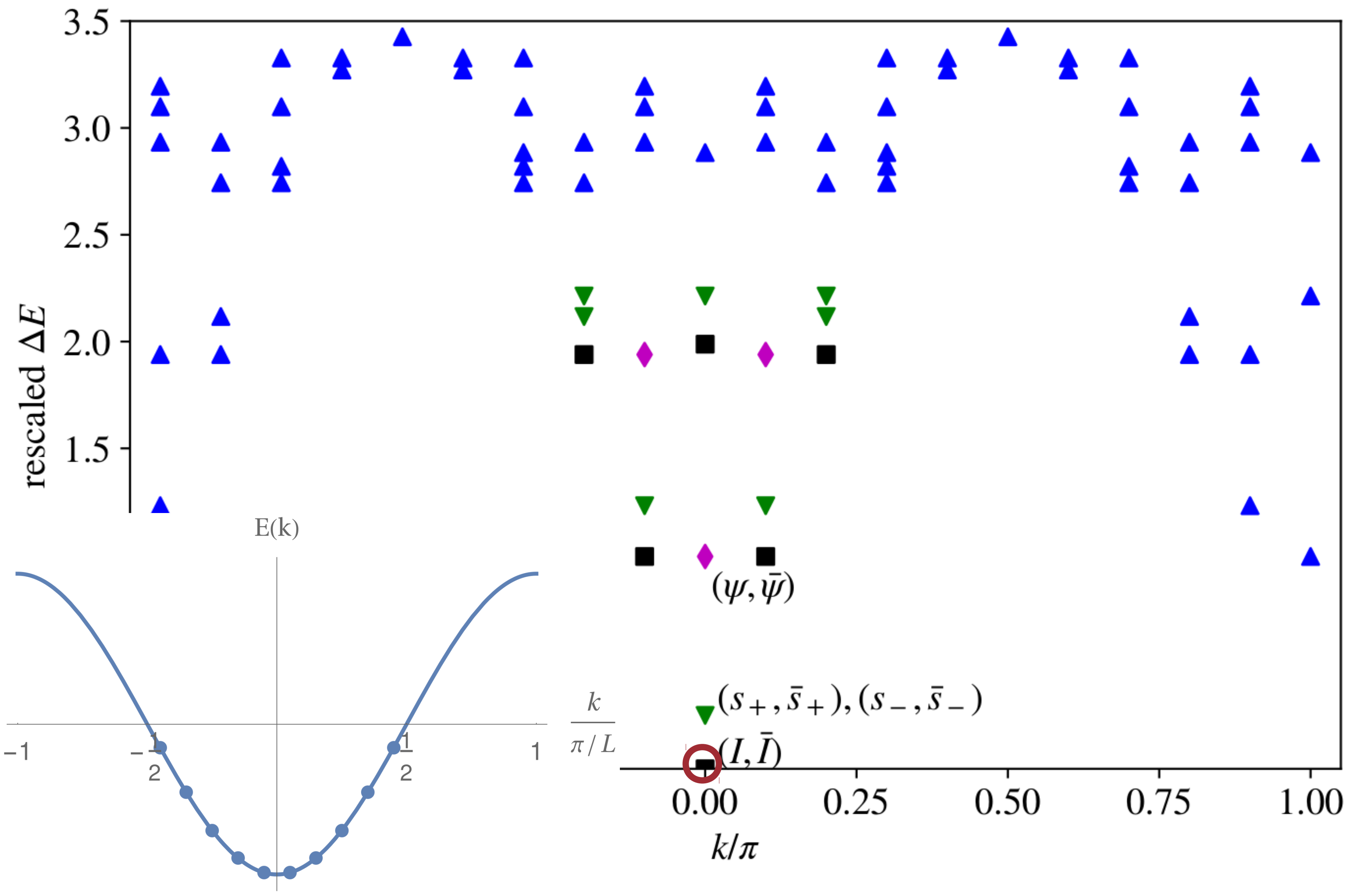

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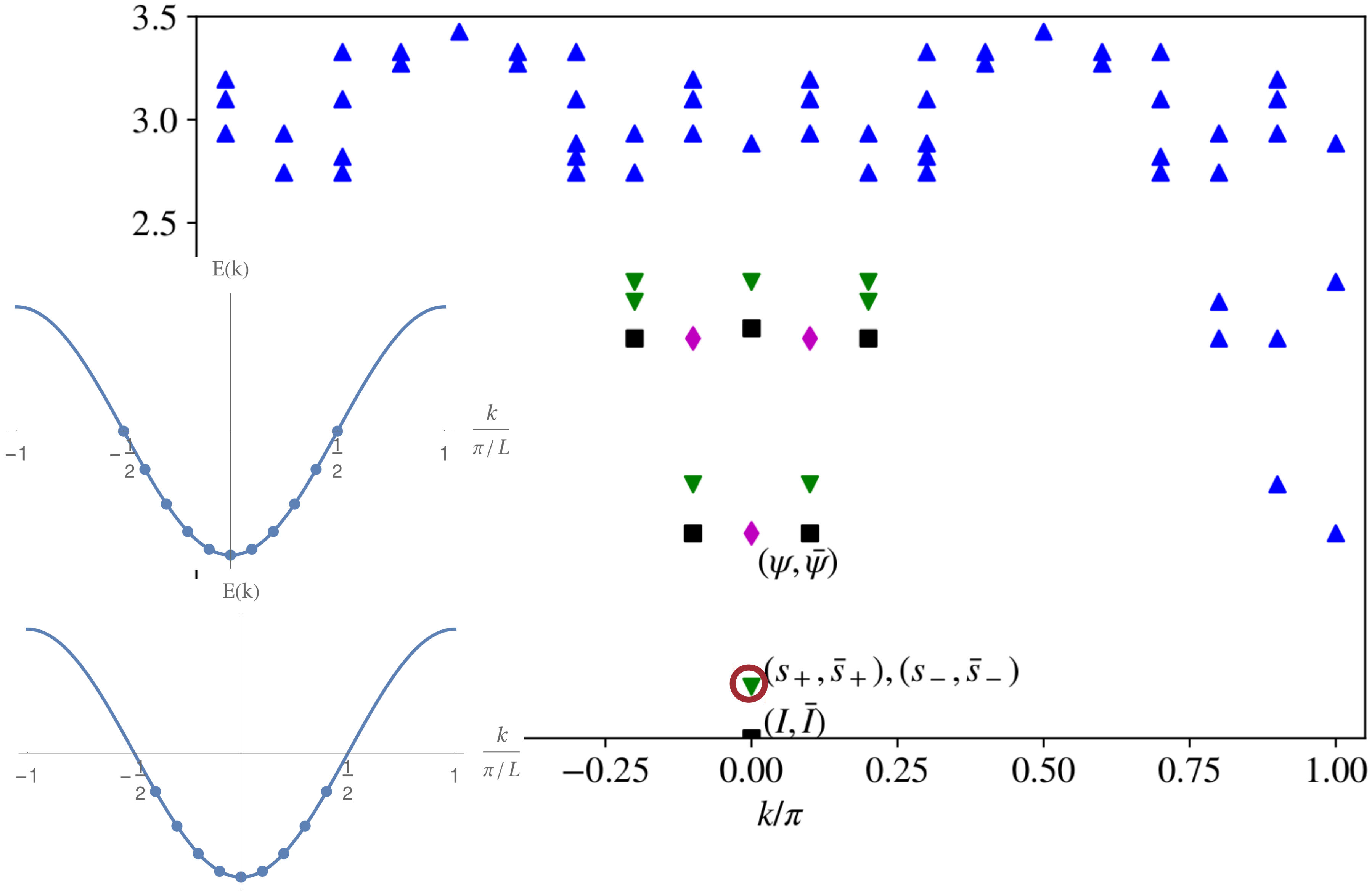

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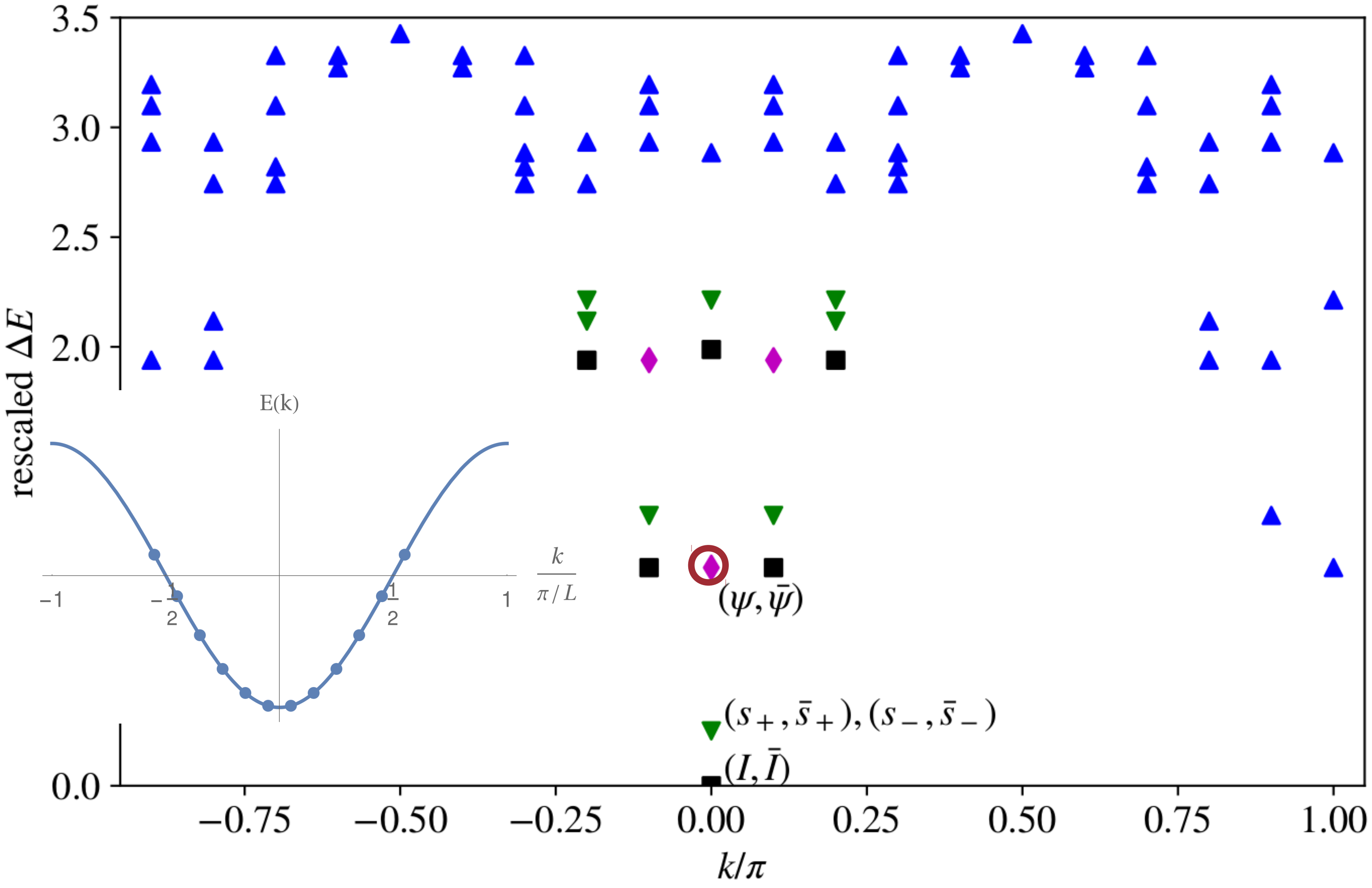

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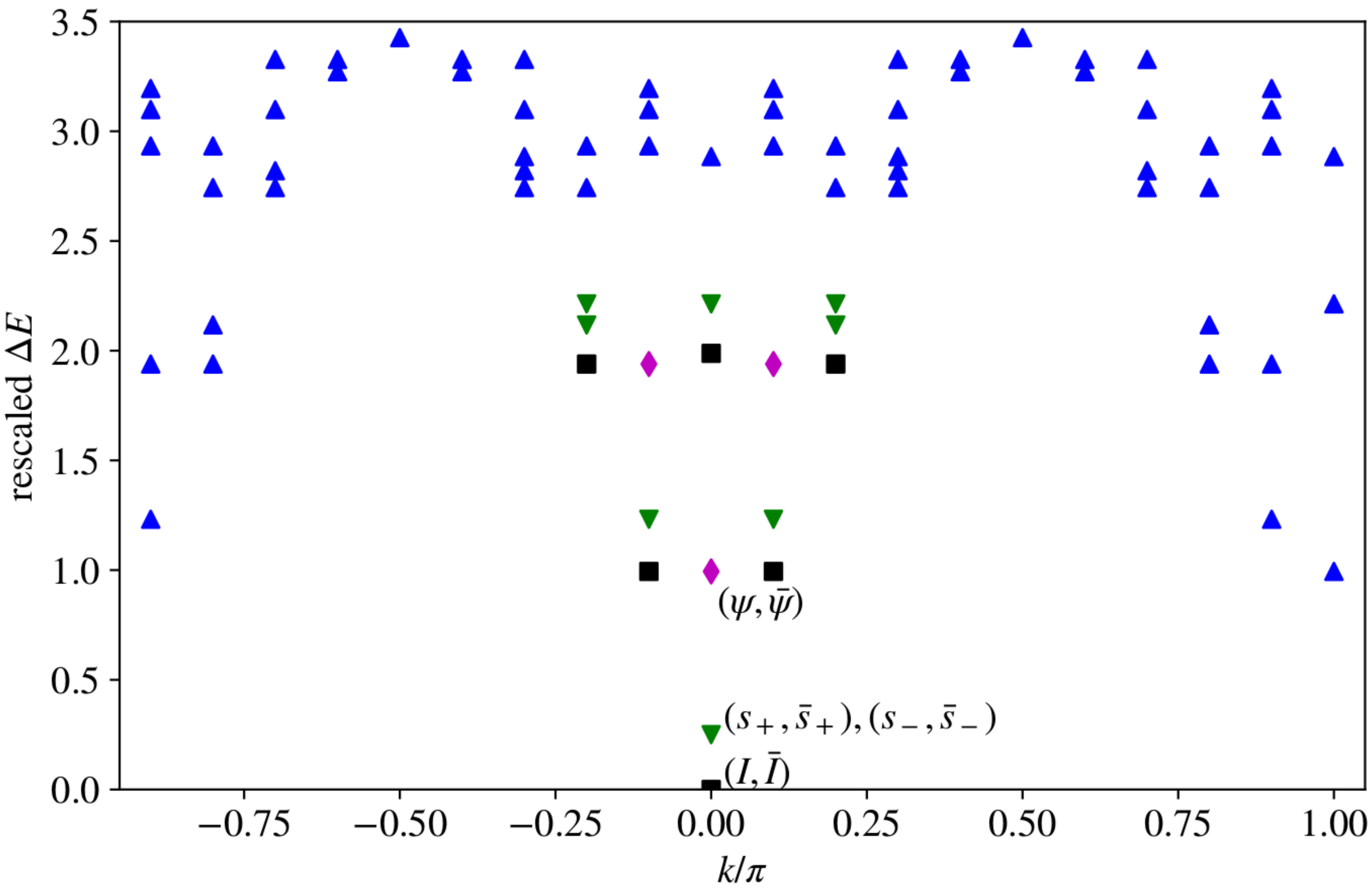


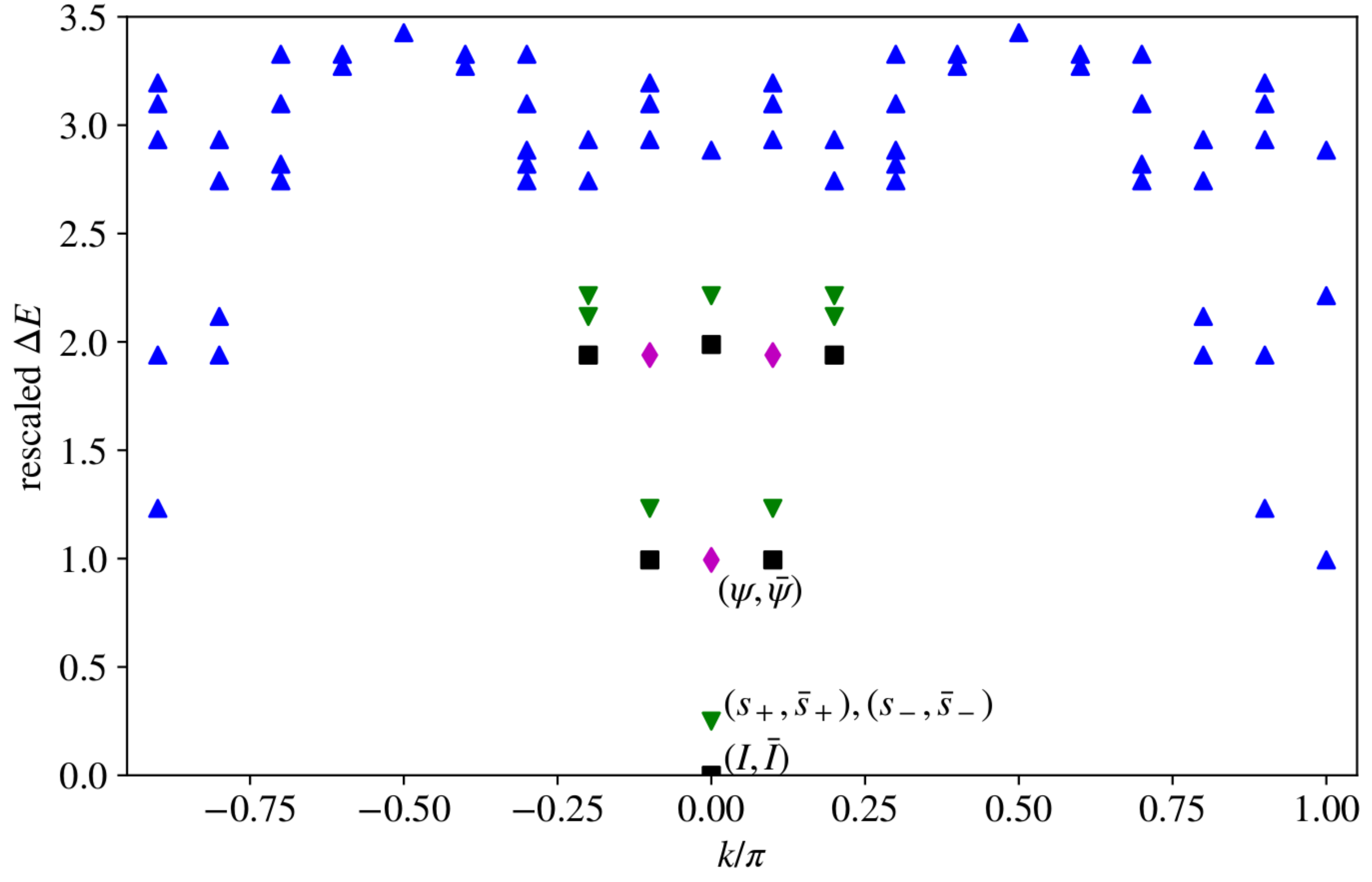
$$d_I = d_{s_+} = d_{s_-} = d_\psi = 1$$

$$\mathcal{D} = \sqrt{1+1+1+1} = 2$$

$$S_{\mathrm{KB}} = \ln \frac{1+1+1+1}{2} = \ln 2$$

Tu, PRL 2017

Spin-1/2 XY chain (rational CFT analysis)



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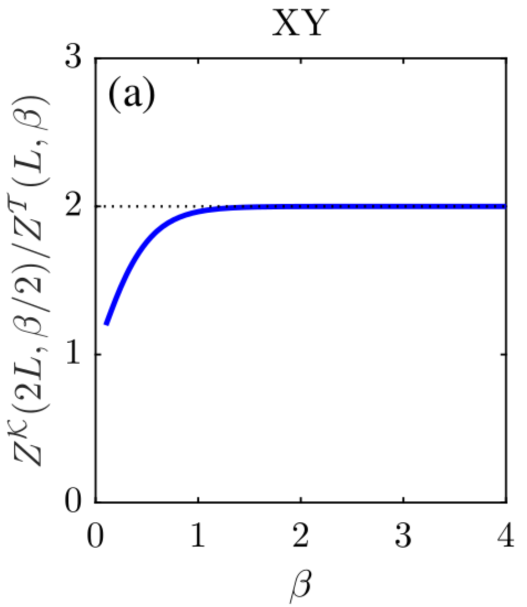
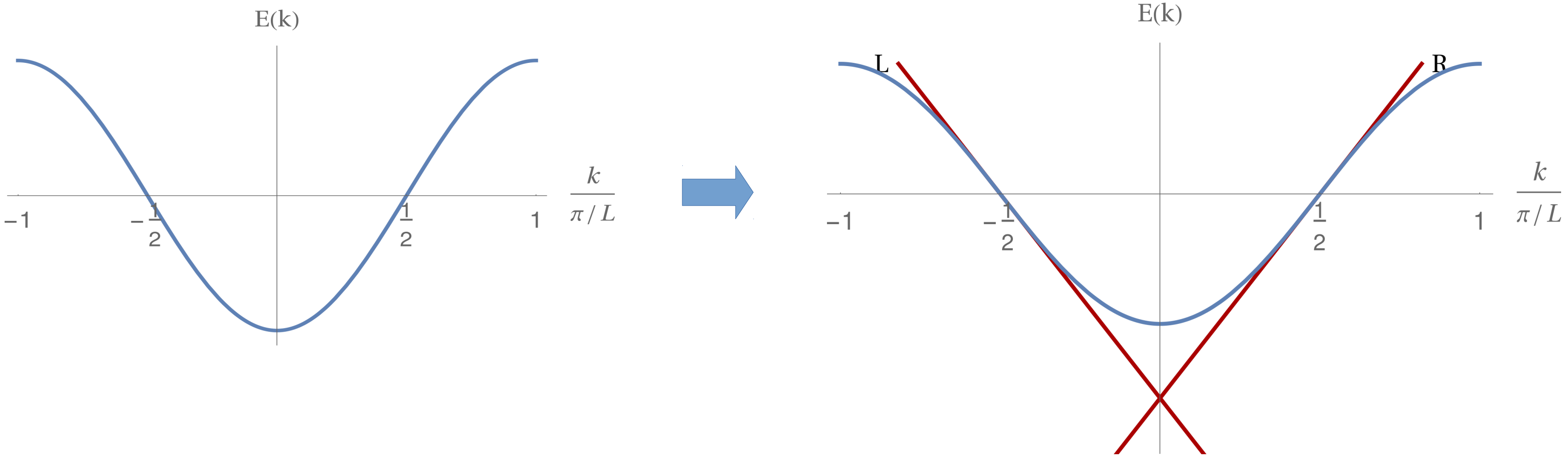


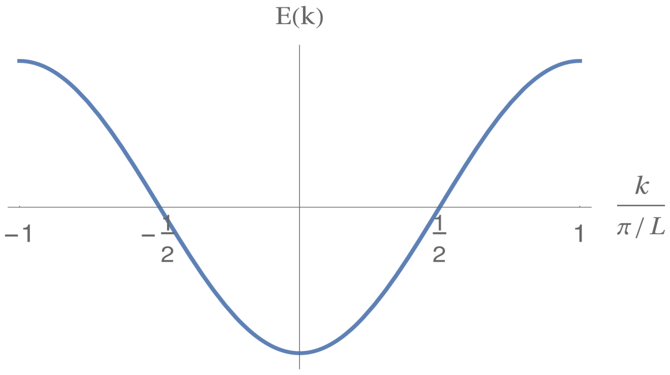
Tu, PRL 2017

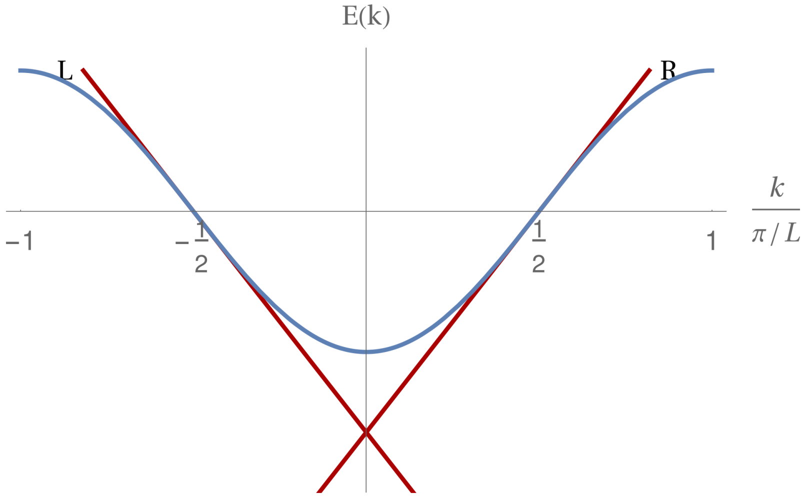
Luttinger liquid description of XY chain



Luttinger Liquid
(spinless, non-interacting)

Luttinger liquid description of XY chain





Luttinger Liquid
(spinless, non-interacting)

The structure of the Hilbert space of LL

Fermi-sea states $|n_L, n_R\rangle$

+

Particle-hole excitations

$$\rho_{\mathrm{L/R},-q} = \frac{2\pi}{L} \sum_k : f^\dagger_{k+q,\mathrm{L/R}} f_{k,\mathrm{L/R}} :$$

Lattice reflection

$n_L \leftrightarrow n_R$

$\rho_{\mathrm{L/R},-q} \leftrightarrow \rho_{\mathrm{R/L},-q}$ (Up to a phase factor)

Spin-1/2 XY chain (compactified boson CFT analysis)

(spinless) Luttinger liquid

↓ bosonization

Compactified free boson

$\phi(x) = \phi(x + L) + 2\pi m R, m \in \mathbb{Z}$

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(spinless) Luttinger liquid

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Bosonization identity $\psi_{\mathrm{L/R}}(x) = a^{-1/2} \mathrm{e}^{\pm \mathrm{i} \frac{\pi}{L} x} F_{\mathrm{L/R}} \mathrm{e}^{-\mathrm{i}\phi_{\mathrm{L/R}}(x)}$

Left- and right-moving boson:

$$\phi_{\mathrm{L/R}}(x) = \sum_{q \neq 0} \frac{\mathrm{i}}{q} \mathrm{e}^{-aq/2} \mathrm{e}^{\mp \mathrm{i} q x} \rho_{q,\mathrm{L/R}} \pm \rho_{0,\mathrm{L/R}} x$$

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Responsible for the discontinuity at the boundary

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Dual fields

$$\phi = \phi_{\mathrm{L}} + \phi_{\mathrm{R}}$$

$$\theta = \phi_{\mathrm{L}} - \phi_{\mathrm{R}}$$

"T-duality"

Bosonization identity $\psi_{\mathrm{L/R}}(x) = a^{-1/2} \mathrm{e}^{\pm \mathrm{i} \frac{\pi}{L} x} F_{\mathrm{L/R}} \mathrm{e}^{-\mathrm{i}\phi_{\mathrm{L/R}}(x)}$

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Responsible for the discontinuity at the boundary

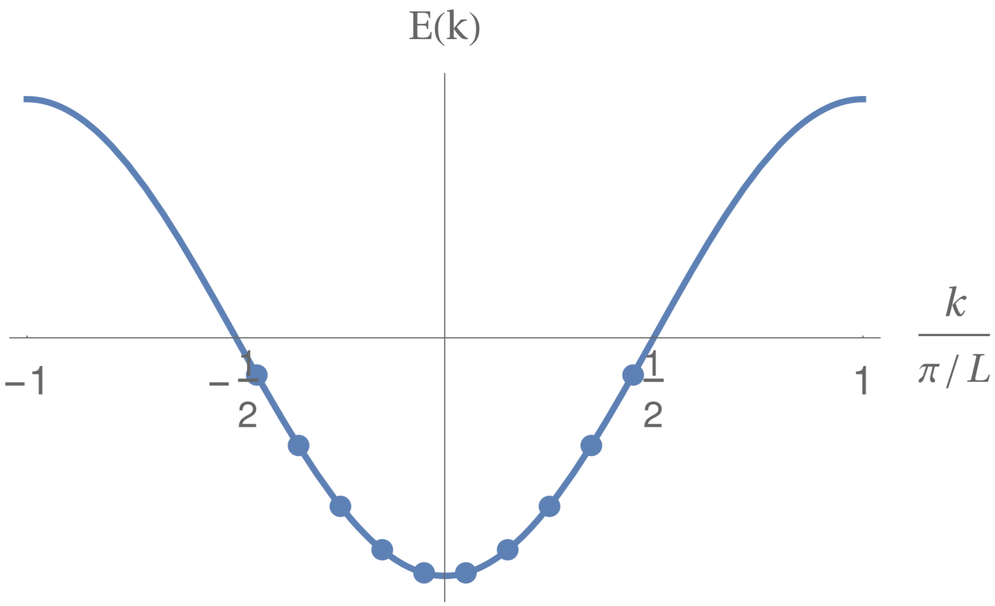
Dual fields

$$\rho_{0,\mathrm{L/R}} = \frac{2\pi}{L} n_{\mathrm{L/R}}$$

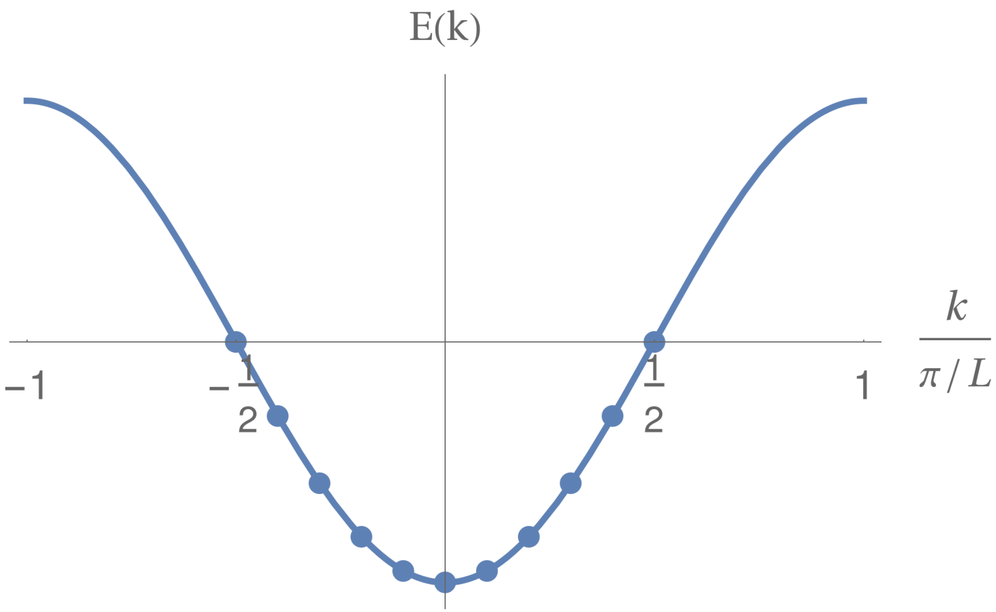
$$\phi = \phi_{\mathrm{L}} + \phi_{\mathrm{R}} \quad \Rightarrow \quad \phi(L) - \phi(0) = 2\pi(n_{\mathrm{L}} - n_{\mathrm{R}})$$

$$\theta = \phi_{\mathrm{L}} - \phi_{\mathrm{R}} \quad \Rightarrow \quad \theta(L) - \theta(0) = 2\pi(n_{\mathrm{L}} + n_{\mathrm{R}})$$

Spin-1/2 XY chain (compactified boson CFT analysis)



NS sector



R sector

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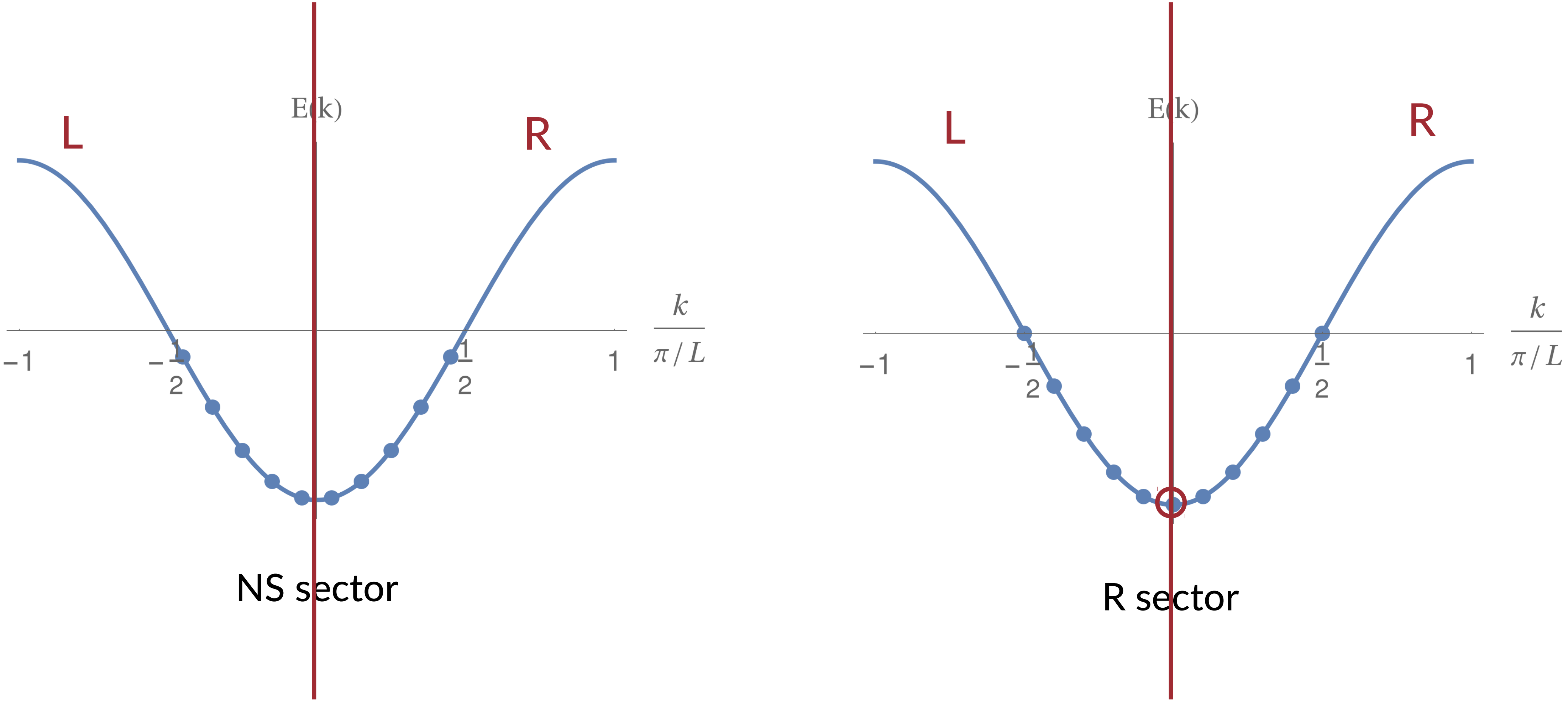

Spin-1/2 XY chain (compactified boson CFT analysis)

(spinless) Luttinger liquid

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Responsible for the discontinuity at the boundary

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➡

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➡

$$R = 2$$
$$R = 1$$

Spin-1/2 XY chain (compactified boson CFT analysis)

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→

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→

$R = 2$ ☺ → $S_{\mathrm{KB}} = \ln R = \ln 2$

$R = 1$

Spin-1/2 XXZ chain

$$H = -\sum_{i=1}^{L}(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + \Delta \sum_{i=1}^{L} S_i^z S_{i+1}^z \rightarrow \Delta \sum_i (f_i^\dagger f_i - \tfrac{1}{2})(f_{i+1}^\dagger f_{i+1} - \tfrac{1}{2})$$

Density-density interaction

Umklapp process

irrelevant for $-1 < \Delta < 1$

$$\phi \Rightarrow \Phi = \sqrt{K}\phi$$

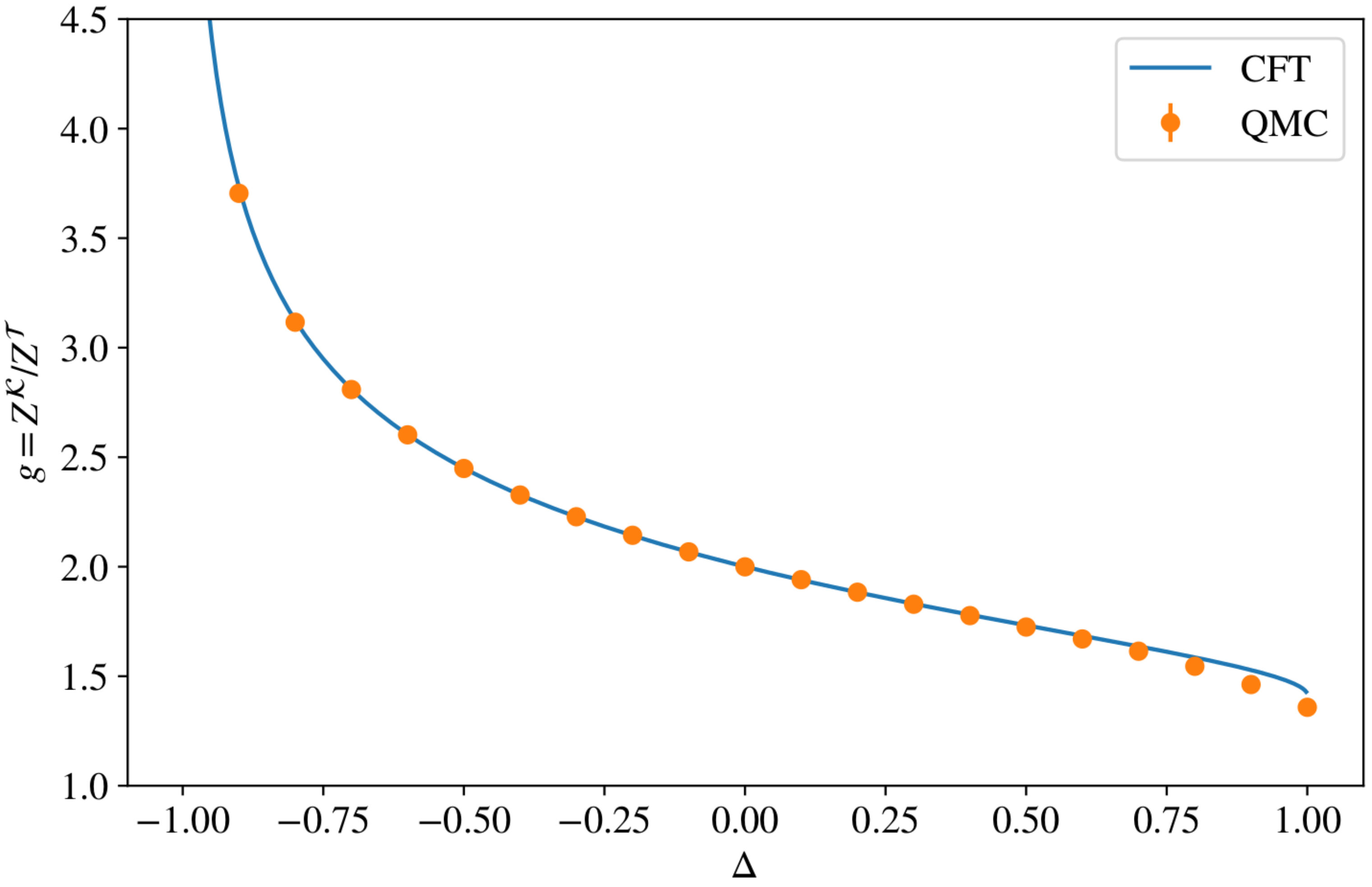
$$\theta \Rightarrow \Theta = \theta/\sqrt{K}$$

$\Rightarrow$

$$S_{\mathrm{KB}} = \ln R = \ln(2\sqrt{K})$$

K Luttinger parameter

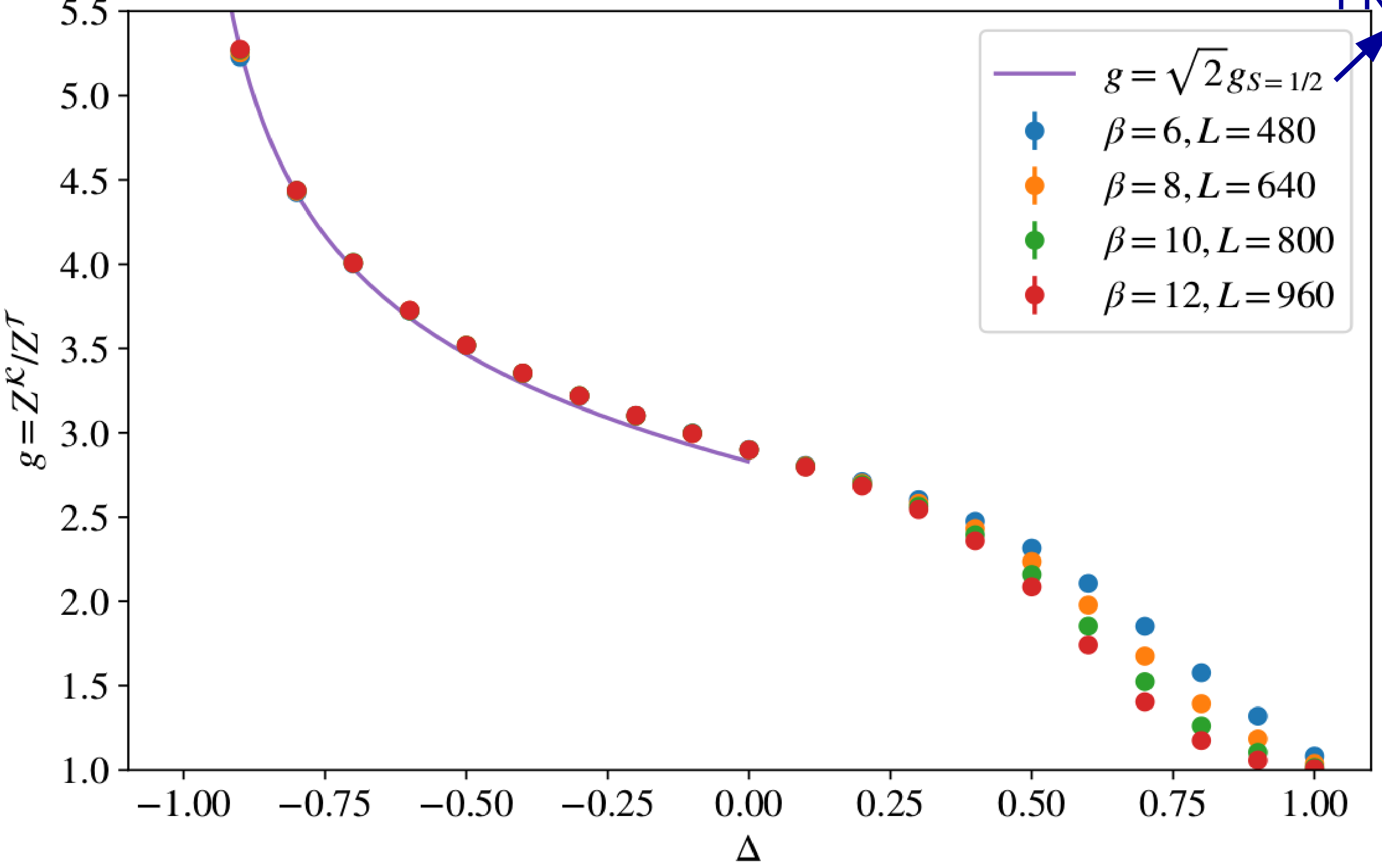
Spin-1/2 XXZ chain



WT, Xie, Wang and Tu, 1805.01300

Spin-1 XXZ chain

Alcaraz and Moreo, PRB 1992



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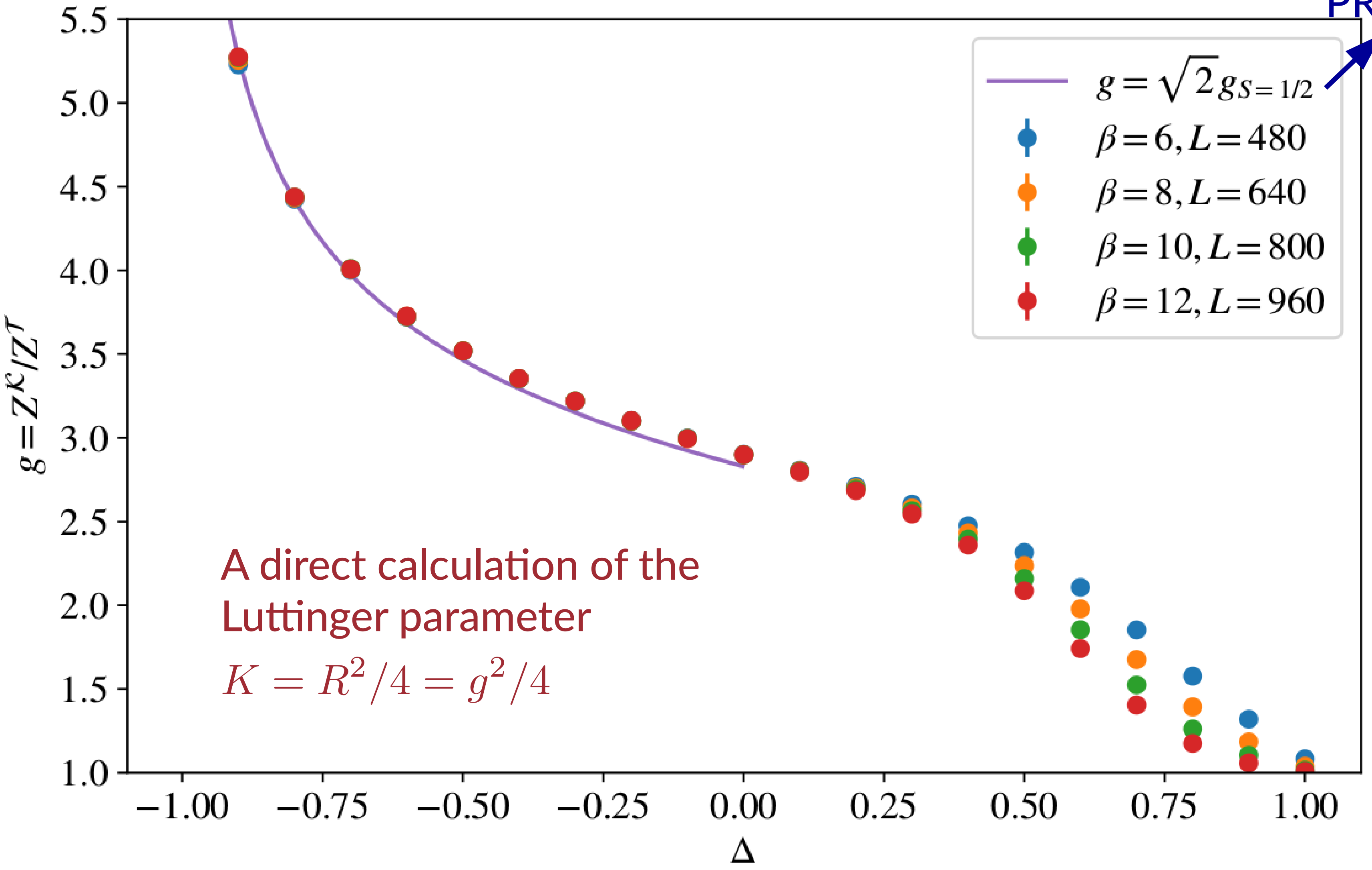

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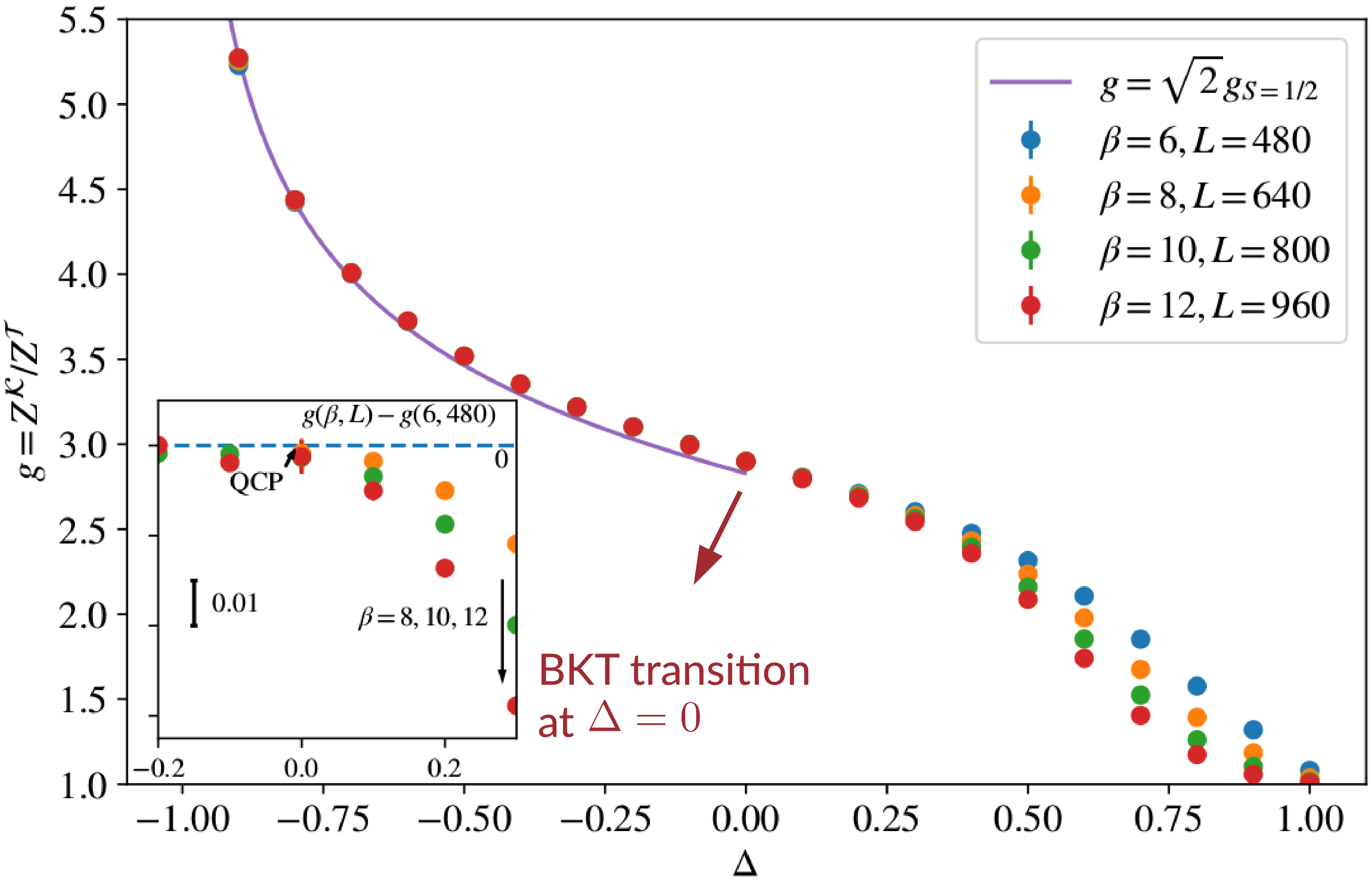

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- The Klein bottle entropy can be a powerful tool to characterize the CFT in conformal critical systems, especially in the rational CFT and the compactified boson CFT, where the Klein bottle entropy can be explicitly calculated

- In lattice models, with a careful definition and analysis of the lattice operation, the Klein bottle entropy can be efficiently extracted by our QMC method

Thank you for your attention!