再谈 notation：

$\vec{r} = (x, y, z)$

谈三维空间中的矢量，当参考零点选定，矢量的 head 位于坐标 (x, y, z) 处

而 $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

(x, y, z)

表示取分量 x, y, z 沿着 $\hat{x}, \hat{y}, \hat{z}$ 方向

$\vec{F} = -\nabla U$

$F_x = -\frac{\partial U}{\partial x}$, $F_y = -\frac{\partial U}{\partial y}$, $F_z = -\frac{\partial U}{\partial z}$

（对应分量相等）

$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

due to $\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1$

$\hat{x} \cdot \hat{y} = \hat{y} \cdot \hat{z} = \hat{x} \cdot \hat{z} = 0$

review:

$$\nabla\varphi = |\nabla\varphi|\,\vec{e}_{\nabla\varphi}$$

$\vec{e}_{\nabla\varphi}$ 是 指是物 φ 增加最快的方向

因为 $\nabla\varphi = \left(\frac{\partial}{\partial x}\varphi, \frac{\partial}{\partial y}\varphi, \frac{\partial}{\partial z}\varphi\right)$

$$\vec{e}_x \frac{\partial}{\partial x}\varphi = \vec{e}_x \frac{\varphi(x+\Delta x, y, z) - \varphi(x, y, z)}{\Delta x}$$

Δx 默认方向是 $\vec{e}_x$ (正向)

$\Delta\varphi$ 为正，即为增加

这样朝 $\vec{e}_x$ 正向，增加方向

$\Delta\varphi$ 为负，结果为

$-\vec{e}_x$ 方向，也是 φ 增加方向

爬山：

$h(x+\Delta x, y+\Delta y) - h(x, y)$

$$\Delta h = \frac{\partial h}{\partial x}\Delta x + \frac{\partial h}{\partial y}\Delta y$$

$\left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}\right) \cdot (\Delta x, \Delta y)$

矢量点乘

$\Delta x^2 + \Delta y^2 = \Delta r^2$

步长，方向

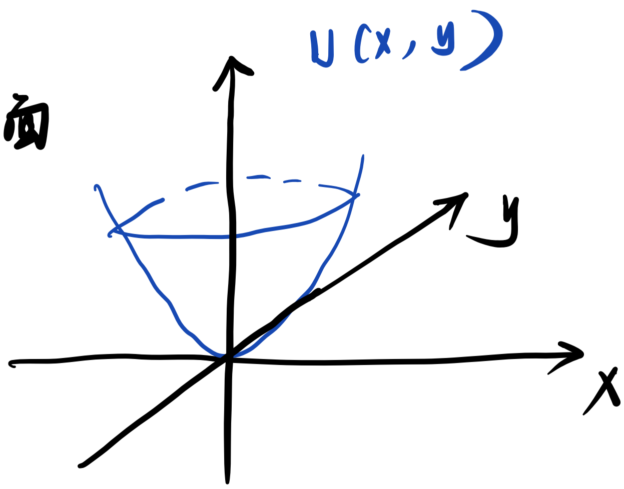
同 $\left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}\right)$

使得 Δh 最大

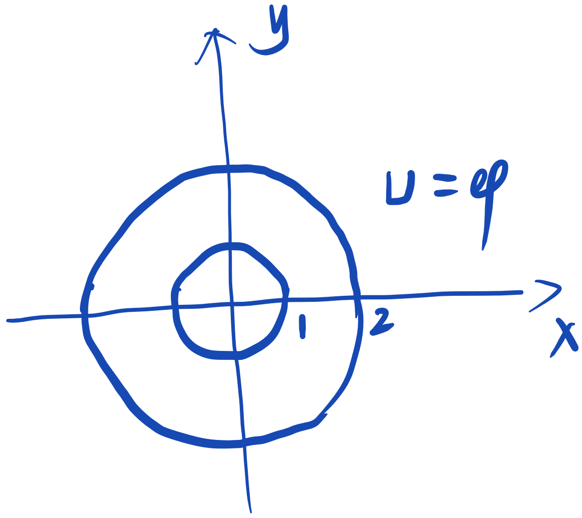
set $\nabla \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$ (in 2d)

$U(x, y) = x^2 + y^2$

等高面



电场线
等高线



求：
∇U 的方向

$$U(B) - U(A) = -\int \vec{F} \cdot d\vec{r}$$

做功原因 $dU = -\vec{F} \cdot d\vec{r} = -F_x\,dx - F_y\,dy$

$$F_x = -\frac{\partial U}{\partial x}$$

$$F_y = -\frac{\partial U}{\partial y}$$

$$\vec{F} = -\nabla U$$

保守力判据： $\dfrac{\partial F_x}{\partial y} = \dfrac{\partial F_y}{\partial x}$

$$\frac{\partial^2 U}{\partial x\, \partial y} = \frac{\partial^2 U}{\partial y\, \partial x}$$

$U(x, y)$ is "smooth"

Conservation of momentum :

若 $\vec{F_e} = 0$ (合外力为0)

$$m_1 \vec{a}_1 = \vec{F}_{12} \qquad \Rightarrow \qquad m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0$$

$$m_2 \vec{a}_2 = \vec{F}_{21} \qquad d(m\vec{v}_1 + m\vec{v}_2)/dt = 0$$

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = 0 \qquad \frac{d\vec{p}}{dt} = 0$$

$$\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$ (mass center)

$$\vec{p} = \sum_i m_i \vec{v}_i = M \frac{d\vec{R}}{dt} = M\vec{V}$$ (质心速度)

$$\vec{F_e} = M \frac{d^2\vec{R}}{dt^2} \qquad \left(M = \sum_i m_i\right)$$

$$m_1 \vec{a}_1 = \vec{F}_{12} + \vec{F}_{e1}$$

$$m_2 \vec{a}_2 = \vec{F}_{21} + \vec{F}_{e2}$$

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = \vec{F}_{e1} + \vec{F}_{e2} = \vec{F_e}$$

$$m_1 \frac{d^2\vec{r}_1}{dt^2} + m_2 \frac{d^2\vec{r}_2}{dt^2} = \vec{F_e}$$

$$\frac{d^2}{dt^2}(m_1\vec{r}_1 + m_2\vec{r}_2) = \vec{F_e}$$

$$\frac{d^2}{dt^2}(M\vec{R}) = \vec{F_e} \Rightarrow \vec{F_e} = M\frac{d^2\vec{R}}{dt^2}$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} = \frac{m_1\vec{r}_1' + m_2\vec{r}_2'}{m_1+m_2}$$

$$\Delta\vec{r}_1 = \vec{r}_1 - \vec{R}$$

$$\Delta\vec{r}_2 = \vec{r}_2 - \vec{R}$$

$$\frac{d\Delta\vec{r}_1}{dt} = \frac{d\vec{r}_1}{dt} - \frac{d\vec{R}}{dt}$$

$$\frac{d\Delta\vec{r}_2}{dt} = \frac{d\vec{r}_2}{dt} - \frac{d\vec{R}}{dt}$$

(质心速度) $\vec{V}$

$$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

$$= \frac{1}{2}m_1(\Delta\vec{v}_1 + \vec{V})^2 + \frac{1}{2}m_2(\Delta\vec{v}_2 + \vec{V})^2$$

$$= \frac{1}{2}m_1\Delta v_1^2 + \frac{1}{2}m_1 V^2 + m_1\Delta\vec{v}_1\cdot\vec{V}$$

$$+ \frac{1}{2}m_2\Delta v_2^2 + \frac{1}{2}m_2 V^2 + m_2\Delta\vec{v}_2\cdot\vec{V}$$

$\Delta\vec{p}$

$$\vec{V}\cdot(m_1\Delta\vec{v}_1 + m_2\Delta\vec{v}_2) = 0$$

质心的动量变化

$$= \frac{1}{2} m_1 \Delta v_1^2 + \frac{1}{2} m_2 \Delta v_2^2 + \boxed{\frac{1}{2}(m_1+m_2) v^2}$$

质心速度不变 ↓

const

能量损失最大，$\Delta v_1 = \Delta v_2 = 0$

相对于质心静止

完全非弹性碰撞，粘在一起

$\Delta v_1 = \Delta v_2 = 0$（质心参考）