rigid body : (what is rigid body)

keep $|\vec{R}_i - \vec{R}_j|$ invariant

many body , not one body (mass point)

what is in common :

θ , w

牛顿力学.

translation	rotation
r	θ
$v = \frac{dr}{dt}$	$w = \frac{d\theta}{dt}$
$a = \frac{dv}{dt}$	$\alpha = \frac{dw}{dt}$
$v^2 - v_0^2 = 2ar$	$w^2 - w_0^2 = 2\alpha\theta$

depend on the distribution of mass

$$E_k = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} \boxed{m_i r_i^2} w^2$$

$\boxed{\text{moment of inertia}}$ $= \sum_i \frac{1}{2} \boxed{I_i} w^2$

common w

Comparison : $\frac{1}{2} m v^2$ $\frac{1}{2} I w^2$

m I

$p = mv$ $L = Iw$

what change the momentum ? the angular momentum ?

$$F = ma = \frac{dp}{dt} \qquad \frac{dL}{dt} = I \frac{dw}{dt}$$

(torque) τ ? $= I \alpha$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \underset{(\vec{v} \times (m\vec{v}))}{\overset{0}{\shortparallel}} + \vec{r} \times m\vec{a} = \vec{r} \times \vec{F}$$

the change of angular momentum : τ

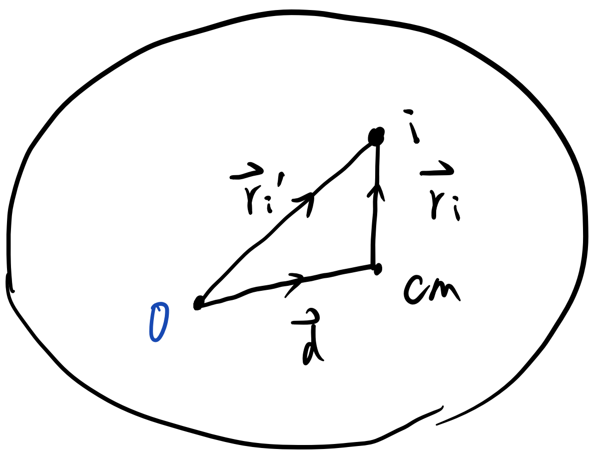
accelerate the rotation

Calculate the moment of inertia

$$I_{end} = \frac{mL^2}{3} \qquad I_{cm} = \frac{mL^2}{12}$$

$$I = I_{cm} + m\left(\frac{L}{2}\right)^2$$

parallel axis theorem



prime : 相对 O

没有 prime : 相对质心 cm

$$I = \sum_i m_i \left(\vec{r}_i' \cdot \vec{r}_i'\right)$$

$$= \sum_i m_i \left(\vec{r}_i + \vec{d}\right) \cdot \left(\vec{r}_i + \vec{d}\right)$$

$$= \sum_i m_i r_i^2 + \sum_i m_i d^2$$

$$+ 2 \underbrace{\sum_i m_i \vec{r}_i}_{=0} \cdot \vec{d}$$

以质心为参考点来度量质心

$$= I_{cm} + md^2$$

the distance between the mass center and the reference point

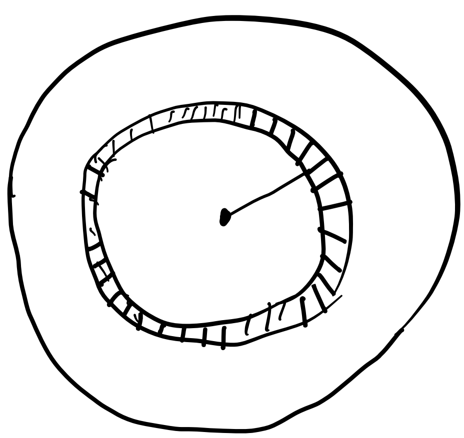
the distribution of the mass. $m(\vec{r})$

like the center of mass

$$\vec{r}_c = \frac{\int \vec{r}\, dm}{m}$$

$$I = \sum_i m_i r_i^2 = \int r^2\, dm$$

圆盘：



$$I = \int_0^R r^2 \, 2\pi r \, dr \, \rho$$

$$= \frac{\pi}{2} R^4 \rho$$

$$= \frac{1}{2} m R^2$$

$\downarrow$

圆环 (ring)

$$I = mR^2$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{M} \text{ (in book)}$$

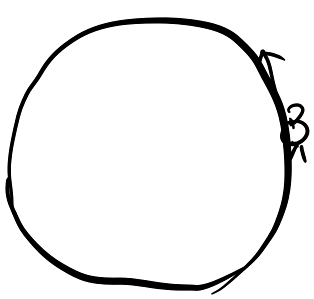
对 $\vec{\tau} = 0$, $\vec{L} = \text{const}$

$$L = I\omega$$

$$\vec{L} = I\vec{\omega}$$

$$\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt}$$

$$= I\alpha$$



$$I_{\text{人}} \omega_{\text{人}} + I_{\text{盘}} \omega_{\text{盘}} = 0$$

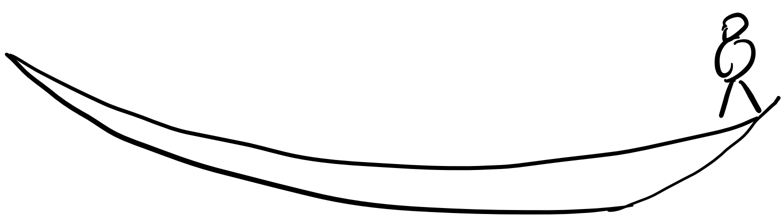
人在圆盘上走一周.

$$I_{\text{人}} \underbrace{\omega_{\text{人}} dt}_{(2\pi-\theta)} + I_{\text{盘}} \underbrace{\omega_{\text{盘}} dt}_{(-\theta)} = 0$$

$$m_{\text{人}} \cancel{R^2} (2\pi-\theta) + \frac{1}{2} m_{\text{盘}} \cancel{R^2} (-\theta) = 0$$

$$\theta = \frac{m_{\text{人}} 2\pi}{m_{\text{人}} + \frac{1}{2} m_{\text{盘}}}$$

类比



船移的距离.

$$\frac{m_{\text{人}} L}{m_{\text{人}} + m_{\text{船}}}$$

花样滑冰运动员

$I_1 w_1 = I_2 w_2$ 角动量守恒

$I_1 \uparrow \quad w_1 \downarrow$

$\frac{1}{2} I_1 w_1^2 - \frac{1}{2} I_2 w^2 \neq 0$

$\vec{F} \cdot d\vec{r} \neq 0$

做功的正负？
能正确找到动能的增量吗？

手的伸长，缩短。

都是 radial direction