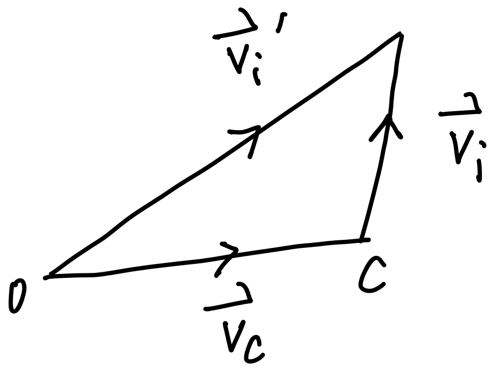
柯尼希定理

(König)



$$\vec{v_i}' = \vec{v_c} + \vec{v_i}$$

Kinetic energy refering to O:

$$\sum_i \frac{1}{2} m_i (\vec{v_i}' \cdot \vec{v_i}') = \sum_i \frac{1}{2} m_i (\vec{v_c} + \vec{v_i}) \cdot (\vec{v_c} + \vec{v_i})$$

$$= \sum_i \frac{1}{2} m_i v_c^2 + \sum_i \frac{1}{2} m_i v_i^2$$

the momentum refer to the mass center reference

$$+ \sum_i \boxed{(m_i \vec{v_i}) \cdot \vec{v_c}}$$

$$(0 = \vec{P_c}) \cdot \vec{v_c}$$

$$= \frac{1}{2} m v_c^2 + \underline{\frac{1}{2} \sum_i m_i v_i^2} \quad (E_{k,in})$$

?

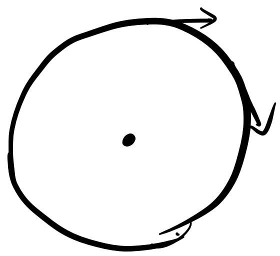
(质心系动能) $= \sum_i m_i r_i^2 w^2$ $= E_k^{cm}$

Q：相对质心只有转动吗？

$$= \underline{\frac{1}{2} m v_c^2} + \frac{1}{2} I_c w^2$$

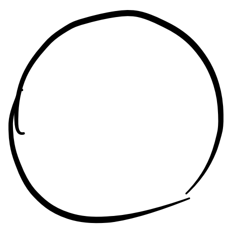
质心的平动动能 　　 绕质心的转动动能

two motions : (can be independent)



the center of mass translation

no friction : $\omega \neq 0$, $V_c = 0$
(rotation)



brake : $V_c \neq 0$, $\omega = 0$

rolling without slipping

骑自行车： $\frac{1}{2} m V_c^2 + \underbrace{\frac{1}{2} I_c \omega^2}_{\frac{1}{2} \frac{mR^2}{2} \frac{v^2}{R^2}}$

$= \frac{1}{2} m V_c^2 + \frac{1}{4} m v^2$

$v = V_c$

转一圈，平动 $2\pi R$

$$E_k = \frac{1}{2} m V_c^2 + \frac{1}{4} m V_c^2$$

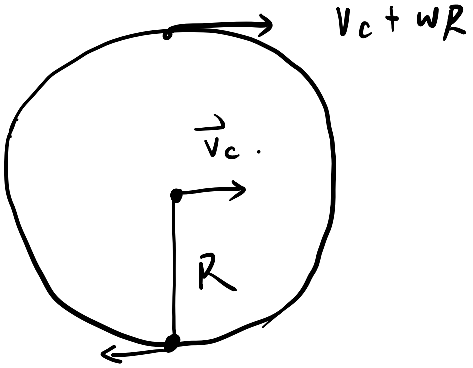
$$= \frac{1}{2}\left(mR^2 + \frac{1}{2} mR^2 \right) w^2 = \frac{1}{2}\left(m + \frac{1}{2}m\right) V^2$$

parallel axis theorem

$E_k = \frac{1}{2} I_{cp} w^2$

视为相对接触点的转动动能

(effective full rotation)



$I_c = \frac{1}{2} mR^2$

$I_{cp} = I_c + mR^2$

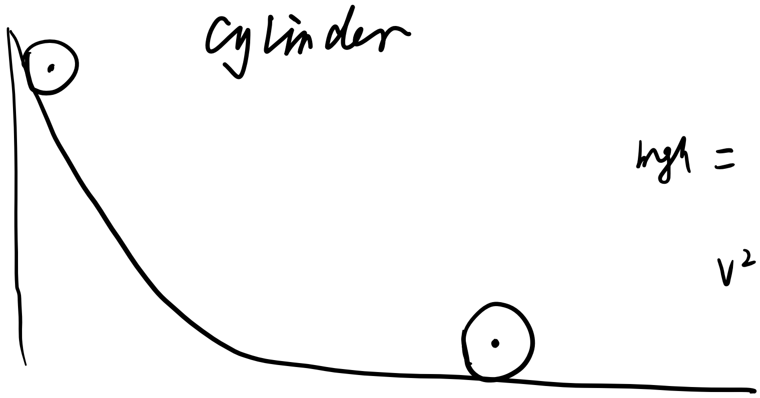
contact point (cp)

it is fixed point

$V_{cp} = V_c - wR = 0$. (reference point: $V_{cp} = 0$)

example :

cylinder



$mgh = \frac{1}{2}\left(m + \frac{1}{2}m\right) v^2$

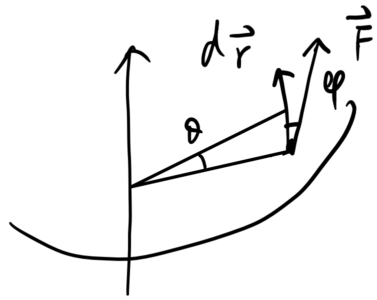
$v^2 = \frac{4}{3} gh \neq 2gh$

转动动能表达式：

hard waving：

$$\frac{1}{2}mv^2 \xrightarrow{v = \omega r} \frac{1}{2}I\omega^2$$

from calculus.



做功微元： $dW = \vec{F}\cdot d\vec{r}$

$$= F\,|d\vec{r}|\cos\varphi$$

$$= \underline{F\cos\varphi}\,R\,d\theta = \underline{F_{tan}}\,R\,d\theta$$

$\vec{\tau} = \vec{R}\times\vec{F}$

$$\boxed{= \tau\,d\theta}$$

$\vec{\tau} = \frac{d\vec{L}}{dt}$

$$= I\alpha\,d\theta$$

$$= I\frac{d\omega}{dt}\,d\theta$$

$\underline{\vec{\tau}\,dt} = d\vec{L}$

冲量矩

$$= I\,\omega\,d\omega$$

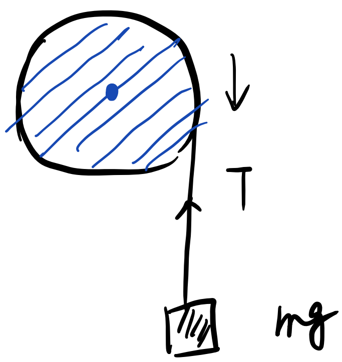
$$= I\,d\omega^2$$

Example

求 a ?

滑轮 (pulley)

translation　　　rotation



$R\alpha = a$

(one string)

受力分析：

$$mg - T = ma.$$

$$R\alpha = a$$

滑轮质量

$$\tau = TR = I\alpha = \frac{1}{2} m R^2 \alpha = \frac{1}{2} m R a.$$

(tangent direction)

$$T = \frac{1}{2} m a$$

$$mg = \left(\frac{1}{2} M + m\right) a.$$

$$a = \frac{mg}{\left(\frac{1}{2} M + m\right)}$$

check it !

from Energy conservation :

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} \left(\frac{1}{2} M R^2\right) w^2$$

$$= \frac{1}{2} m v^2 + \left(\frac{1}{4} M \boxed{R^2 w^2} = \frac{1}{4} M v^2\right)$$

$$2ah = v^2$$

$$mg \frac{\cancel{v^2}}{2a} = \frac{1}{2} m \cancel{v^2} + \frac{1}{4} M \cancel{v^2}.$$

$$a = \frac{mg}{2\left(\frac{1}{2} m + \frac{1}{4} M\right)} = \frac{mg}{m + \frac{1}{2} M}$$