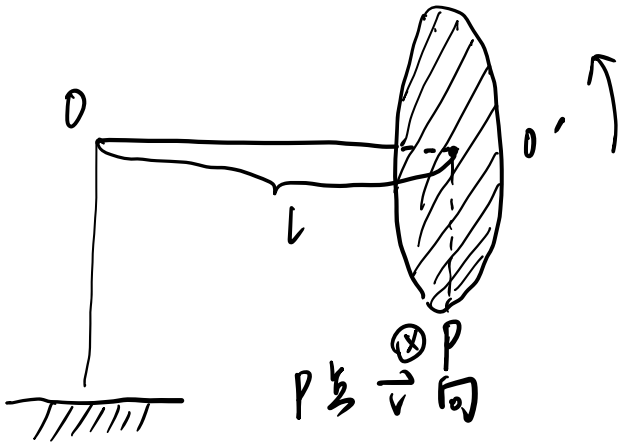
/'dʒairoskoup/

陀螺垂直于纸面



$\vec{F}=m\vec{g}$

$\vec{\tau}$

$\tau = mgL$

⊗ 重力矩

如果陀螺 without spinning

将因为重力矩顺时针旋转往下掉

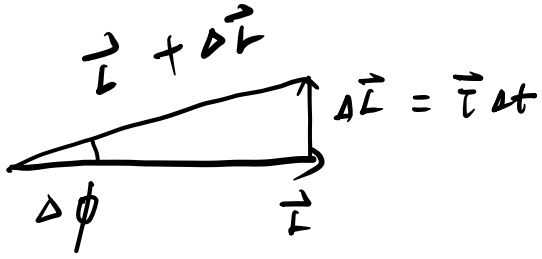
如果陀螺从 OO' 方向逆时针旋转，

有 $\vec{w} \parallel \vec{OO'}$

$\vec{L} = I\vec{w}$ 同样指向

$\tau\, \Delta t = \Delta L$

如果将 $\vec{L}$ 翻到平面的竖直方向，则有



rotating

不会掉下来

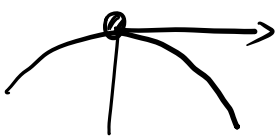
$$\Delta L = L\Delta\phi$$

$$\tau = \frac{\Delta L}{\Delta t} = L\frac{\Delta\phi}{\Delta t} = L\omega_p$$

$$\omega_p = \tau/L = mgl/I\omega$$

ω：表圆盘沿 OO' 的 spinning

ω_p：圆盘沿 ⊗ 方向的进动



没有 $\vec{v}$ 受重力

直接往下掉，

有 $\vec{v}$ 将开始转

如果 $mg = mv^2/r$

v 的大小将不改变

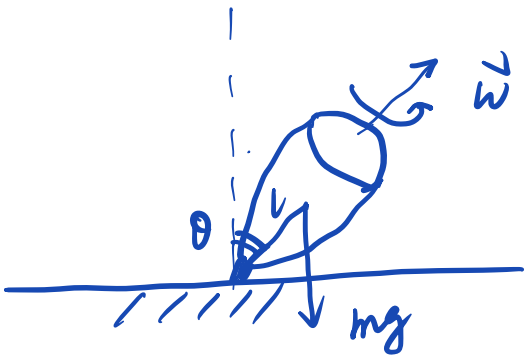
look back to the eqn.

$$\omega_p = \frac{mgL}{I\omega}$$

$$\omega_p \omega = \frac{mgL}{I} = \text{const}$$

refer to 打陀螺.

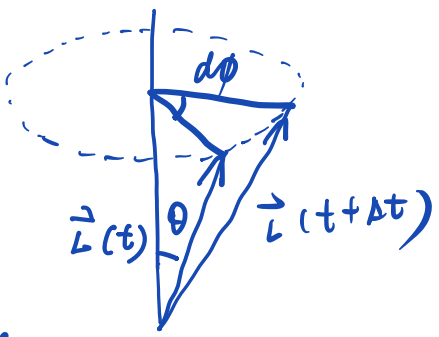
precession of a rotating top



垂直方向

$\vec{L}_\perp$ 单未改变

θ is fixed



$$|d\vec{L}| = L\sin\theta\, d\phi$$

$$= L\sin\theta\, \omega_p\, dt$$

$$\left|\frac{d\vec{L}}{dt}\right| = L\omega_p\sin\theta$$

$$L = I\omega$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = mgL\sin\theta$$

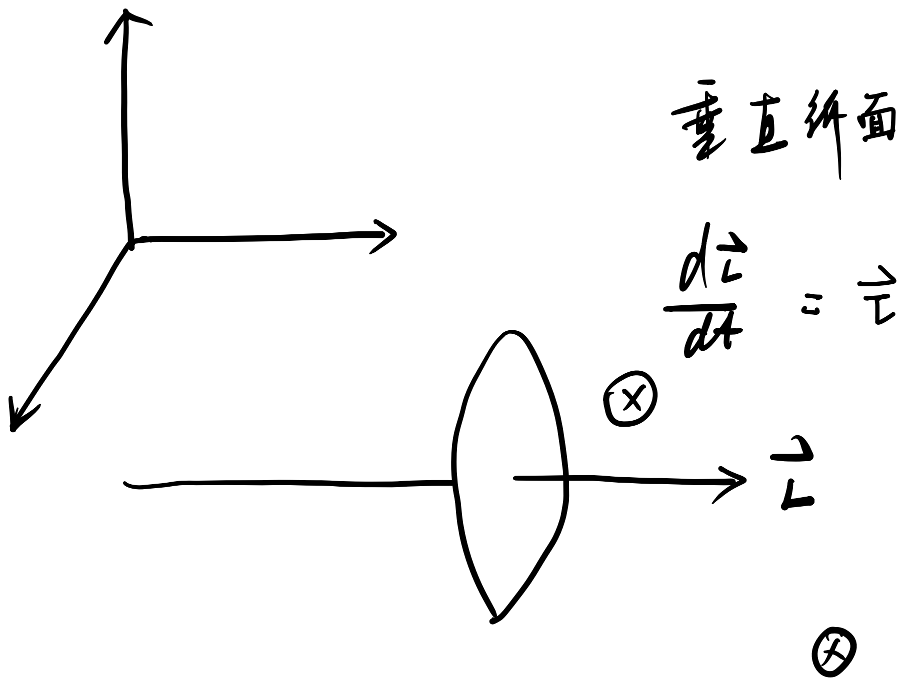
$\omega_p \omega = \text{const}$

陀螺倒下前 $\omega_p \uparrow$

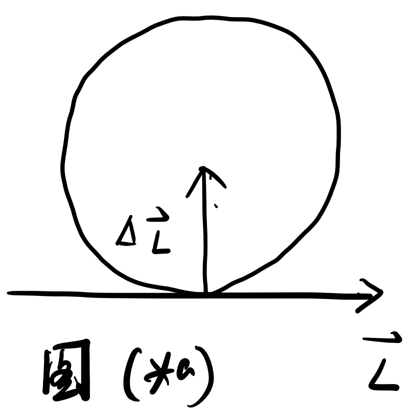
$\omega \downarrow$

$\omega_p = mgL/I\omega$ 与θ无关

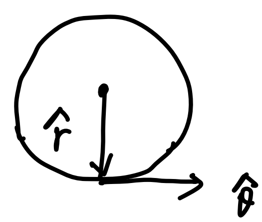
比较思考



垂直于纸板平面转到
平面上来



($\tau = \omega$)



类比圆周运动

$$\vec{F} = \frac{d\vec{p}}{dt} = p\, d\hat{\varphi}/dt = p\frac{d\hat{\varphi}}{dt} = -p\omega\hat{\varphi}$$

比较匀速圆周运动

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d(L\hat{\tau})}{dt}$$

$\hat{\varphi} = \hat{\varphi} = \hat{\varphi}$

(切向)

$$= \frac{dL}{dt}\hat{\tau} + L\frac{d\hat{\tau}}{dt}$$

$\hat{\tau} = \hat{\varphi}$ (here)

$$= \frac{dL}{dt}\hat{\tau} - L\frac{d\theta}{dt}\hat{r}$$

(图 * a)

$$= \frac{dL}{dt}\hat{\tau} - L\omega\hat{r} \quad (*)$$

$\frac{dL}{dt} \neq 0$, 角动量大小改变

如果只实现方向改变

$$\vec{\tau} = -L\omega\hat{r}$$

↓

只实现方向改变，力矩与角动量之间有更

好的匹配关系，否则将改变 L

正如维持圆周运动的向心力一样

$$\vec{F} = -p\omega\hat{r}$$