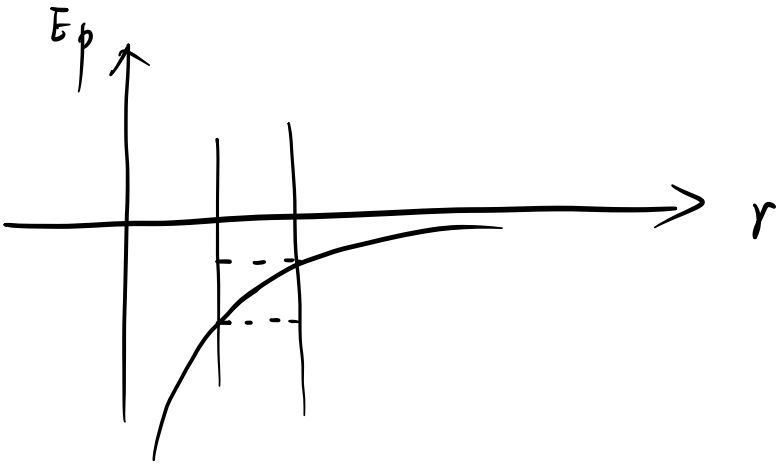
Kepler's three laws

$$E_p = -\frac{GMm}{r}$$

$$E_p = mgh$$

矛盾吗?



$$E_p = -\frac{GMm}{(R_E + h)}$$

$$= -\frac{GMm}{R_E}\ \frac{1}{(1+\frac{h}{R_E})}$$

$$= \sim \left(1 - \frac{h}{R_E} + \cdots\right)$$

$$= -\frac{GM_m}{R_E} + \boxed{\frac{GM_m}{R_E^2}} h \quad (= mg)$$

I. Each planet moves around the sun in an ellipse, with the sun at one focus.

II. The radius vector from the sun to the planet sweeps out equal areas in equal intervals of time.

III. The squares of the periods of any two planets are proportional to the cubes of the semimajor axes of their respective orbits: $T \propto a^{3/2}$.

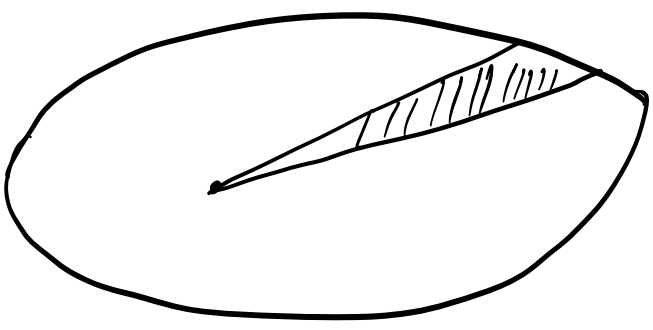
1st Law : ellipse

轨迹在以太阳为焦点的椭圆

平面上

2nd Law : $\frac{dA}{dt} = \text{const}$

扫过的面积速度相等



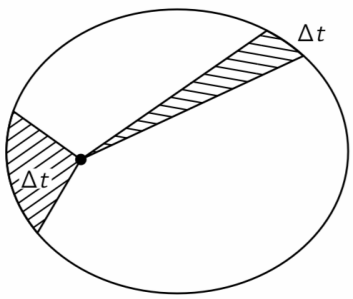
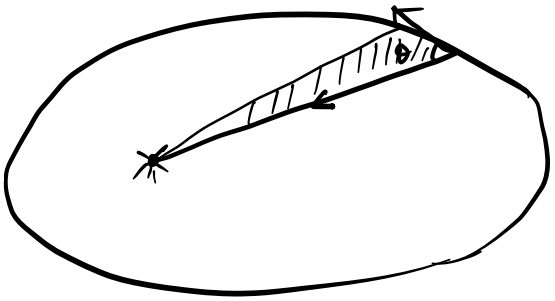


Fig. 7-2. Kepler's law of areas.



$$\vec{\tau} = \vec{r} \times \vec{F} = 0$$

$$\vec{L} = \vec{r} \times \vec{p} = c$$

$$\frac{r\, m v \sin\theta \; dt}{}$$

A line drawn from the sun to a planet sweeps out equal areas in equal times

$$\frac{\boxed{r\, v\, dt \sin\theta}}{2}$$

$$\boxed{2\, ds}\, m = c\, dt$$

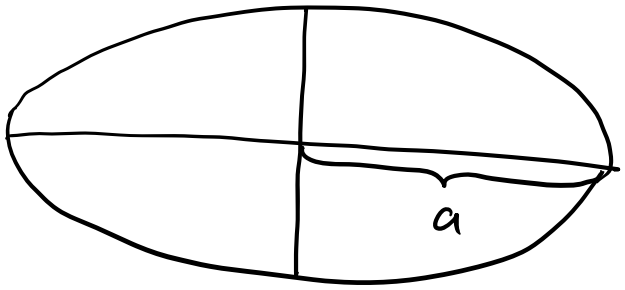
$$\frac{ds}{dt} = \frac{c}{2m} = \text{const}$$

掃角速度

Kepler's 2nd Law

Central force

3rd Law : $\frac{T^2}{a^3} = \text{const}$



if it is circle

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = m\frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$\Rightarrow$$

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$\frac{T^2}{r^3} = \text{const} = \frac{4\pi^2}{GM}$$

(1546 ~ 1601) danish astronomer

Tycho : rich guy, build lab

kepler : assistant

(1571 ~ 1630) 40 years' work

Newton : (1643 - 1727)

$$E = E_p + E_k$$

$$= -\frac{GMm}{r} + \frac{1}{2}mv^2$$

$$= -\frac{GMm}{r} + \frac{1}{2}m\frac{v^2}{r}r \quad (\text{圆周运动})$$

$$= -\frac{GMm}{r} + \frac{1}{2}\frac{GMm}{r}$$

$$= -\frac{1}{2}\frac{GMm}{r}$$

$E < 0$

系统总势负是不能到达无穷远处的

If then, $E_p = 0$, $E_k < 0$

如何逃逸地球呢？

$E > 0$

$E = 0 \Rightarrow$

$$E_K = \frac{1}{2} m v^2 = \frac{GMm}{r} = -E_p$$

for the earth, $r = R_E$

$$v = \sqrt{\frac{2GM}{r}}$$

以这样的速度发射将不再回来，

v_1 : for the earth (~ 11.2 km/s)

v_2 : for the sun (~ 617.7 km/s)

v_3 : for the Galaxy

. . . .

for black hole ($\sim 3 \times 10^5$ km/s)

(what's the meaning?)

第一，二，三 escape velocity

第一宇宙速度书上一般称为贴地圆周运动

$$V = \sqrt{\frac{Gm}{r}} \quad (\text{并未逃逸})$$

V_1 ：俗称、第二宇宙速度

Q： 月球上有大气层吗？