simple harmonic motion

1° 弹簧 $F = -kx$

$$F = ma = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

trial solution $\omega_0 = \sqrt{\frac{k}{m}}$

Q：将弹簧竖直着会怎样？

$$x_1 = \cos\omega_0 t \rightarrow \cos(\omega_0 t + \phi)$$

$$x_2 = \sin\omega_0 t \rightarrow \sin(\omega_0 t + \phi)$$

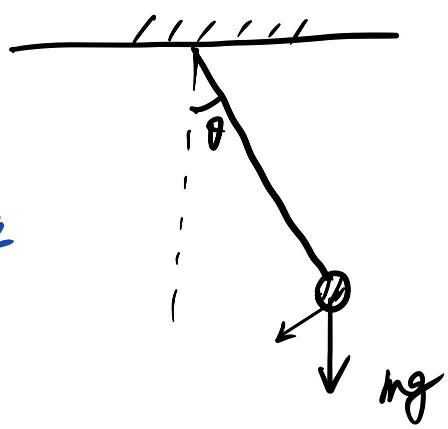
$$x = Ax_1 + Bx_2$$

linear superposition

如果用 $x = Ae^{i\omega_0 t} + Be^{-i\omega_0 t}$ 组合，则有 $A = B^*$

x 是实数

2° simple pendulum



tangent force: $-mg\sin\theta$

此处可用力矩

$$-mgl\sin\theta = I\alpha = ml^2\frac{d^2\theta}{dt^2}$$

θ 的正方向是逆时针

$$-mg\sin\theta = ma = m\frac{d^2\vec{r}}{dt^2}$$
$$= m l \frac{d^2\theta}{dt^2}$$

$$\theta \to 0$$

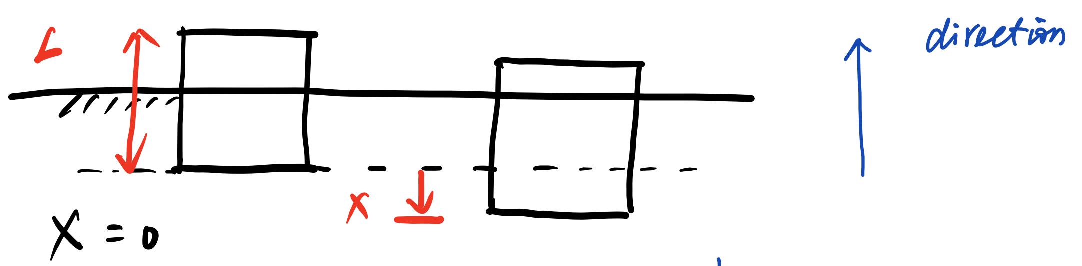
$$m l \frac{d^2\theta}{dt^2} + mg\,\theta = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

$$w_0 = \sqrt{\frac{g}{l}}$$

$$T = \frac{2\pi}{w_0} = 2\pi\sqrt{\frac{l}{g}}$$

3° floating block



buoyancy /ˈbɔɪənsi/
浮力

$X = 0$: $\vec{F}_b + \vec{F}_g = 0$

x处 : $\vec{F}_b + \vec{F}_g = \rho_l g(-x)A \quad \propto A$

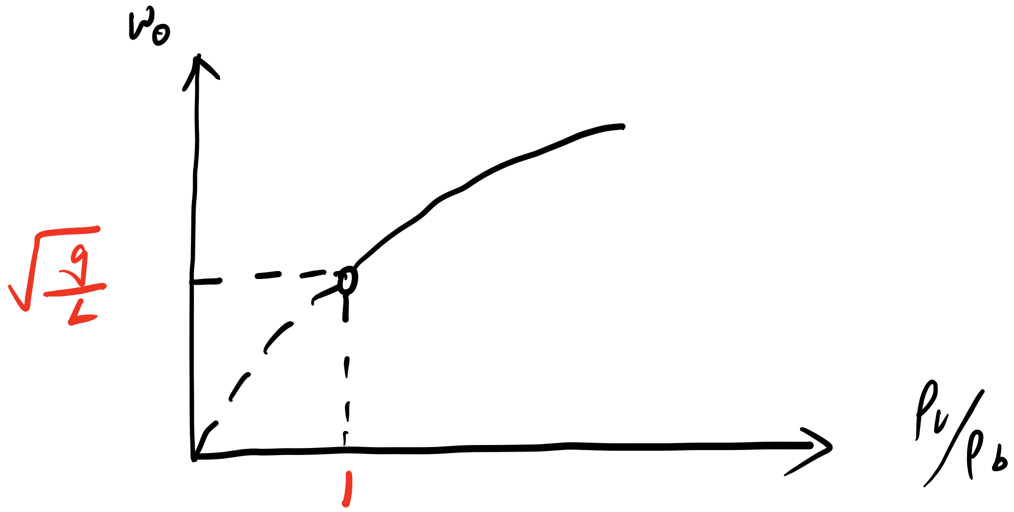
A ↓ bottom surface

EOM : eqn of motion

$$\rho_L g(-x)\cancel{A} = \rho_b \cancel{A} L a = \rho_b \cancel{A} L \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \frac{\rho_L}{\rho_b}\frac{g}{L}x = 0$$

$$w_0 = \sqrt{\frac{\rho_L}{\rho_b}\frac{g}{L}}$$



what about $\rho_L/\rho_b < 1$?

I. Conserved energy :

$\frac{dx}{dt}$

$$X = A\cos(w_0 t + \phi)$$

$$V = -Aw_0\sin(w_0 t + \phi)$$

$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\frac{1}{2} m w_0^2 A^2 \cos^2(w_0 t + \phi)$$

$$\frac{1}{2} m A^2 w_0^2 \quad \sin^2(w_0 t + \phi)$$

$$= \frac{1}{2} m A^2 w_0^2 = \text{const}$$

$$E_p + E_k = E$$

Although the conservation of energy is <u>NOT</u> as powerful as Newton's EOM, it remains valid even when Newtonian mechanics fails ^^

II. SHM in <u>phase space</u>

$\equiv$ x - p space

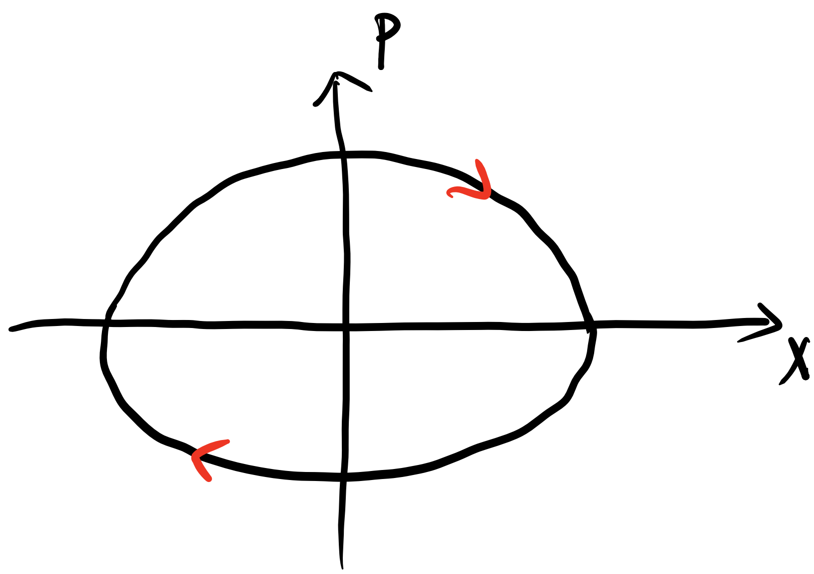
$$\frac{p^2}{2m} + \frac{1}{2} k x^2 = E \Rightarrow$$

$$\frac{p^2}{2mE} + \frac{x^2}{2E/k} = 1$$

the trajectory in the phase space is an ellipse !

$$\left(\frac{p}{p_0}\right)^2 + \left(\frac{x}{x_0}\right)^2 = 1$$

$$p_0 = \sqrt{2mE}, \quad x_0 = \sqrt{2E/k}$$



$$\oint p\,dx = ?$$

area of the ellipse

$$= \pi p_0 x_0$$

$$= \pi \sqrt{2mE} \sqrt{2E/k}$$

$$= 2\pi E / w_0$$

classical mechanics : energy change continously

quantum world :

$$\oint p\,dx = (n + \tfrac{1}{2}) h$$

$n = 0, 1, 2, 3, \cdots$ quantized

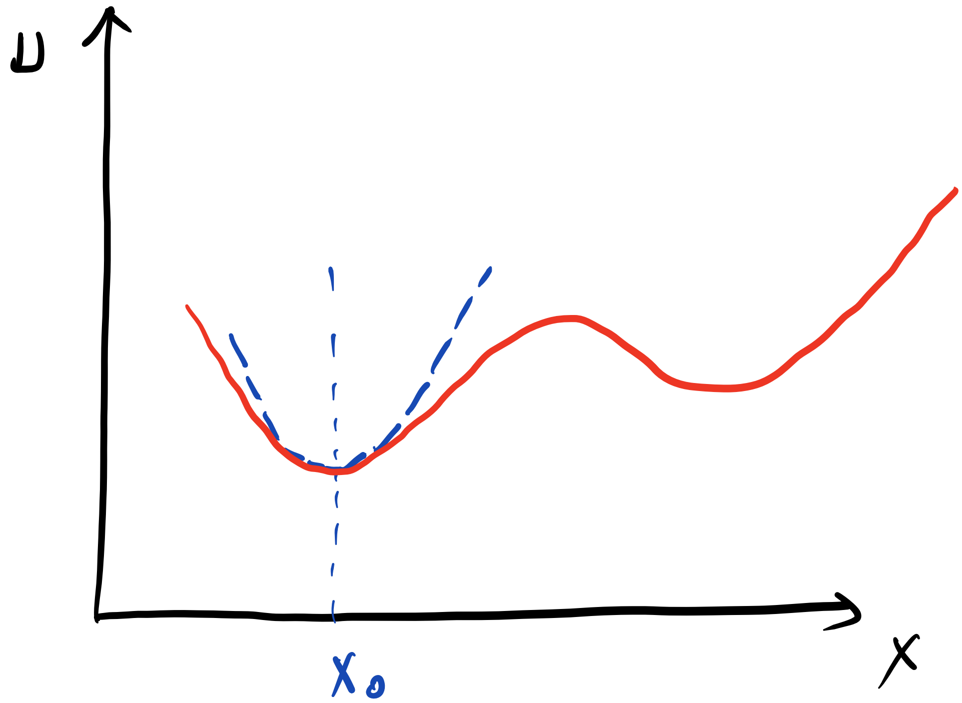
$$2\pi E / w_0 = (n + \tfrac{1}{2}) h$$

$$E = (n + \tfrac{1}{2}) \hbar w_0 \qquad \hbar \equiv \frac{h}{2\pi}$$

$\frac{1}{2} \hbar w_0$: zero - point energy

Ⅲ. why simple harmonic motion is so common in nature?

potential profile



$$U(X) = U(X_0) + \frac{1}{2} k (X - X_0)^2$$

$$F(X) = -\frac{dU(X)}{dX}$$

$$= -k(X - X_0)$$

Hooke's law again!

X_0 : the stable equilibrium point

as long as $(x - x_0)$ is small,

the dynamics is roughly

$$x - x_0 = A \cos(\omega_0 t + \phi)$$

— SHM is <u>everywhere</u> ! !