Boltzmann distribution

two constraints:

$$\sum_i N_i = N$$

$$\sum_i N_i E_i = E$$

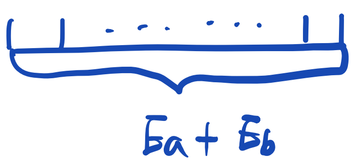
detailed balance

$$p\left((a,b) \to (a',b')\right) = p\left((a',b') \to (a,b)\right)$$

$$\downarrow$$

$$\frac{N_a}{N}\frac{N_b}{N}\frac{2}{E_a+E_b} = \frac{N_{a'}}{N}\frac{N_{b'}}{N}\frac{2}{E_{a'}+E_{b'}}$$

(能级等间距)



$$E_a+E_b = E_{a'}+E_{b'}$$

两个粒子发生碰撞，在总能量不变

的约束下 可变成任何允许的能量

detailed balance（即参与求和的任何能级）

细致平衡条件苛刻，达成 balance 可以由

(a, b)，(a', b') 作和的进出平衡。

“流守恒”

$$N_a N_b = N_{a'} N_{b'}$$

$$\log N_a + \log N_b = \log N_{a'} + \log N_{b'} \quad ①$$

$$E_a + E_b = E_{a'} + E_{b'} \quad ②$$

试探解 $\log N_a = AE_a + C_1$ 猜的

满足 ① ②

$$N_a = e^{AE_a} + C_2$$

$$\frac{N_a}{N} \sim e^{AE_a} + C_2$$

distribution

$$\sum_a e^{AE_a} = 1$$

$$A = -E/N$$

→ Continuous

$$\int_0^\infty e^{-x}\,dx = 1$$

$\frac{1}{\tau} e^{-m/\tau}$: probability

$$\frac{1}{\tau}\int_0^\infty e^{-m/\tau}\,dm = 1 \quad \left(\sum_i p_i = 1\right)$$

$$\tau = \frac{1}{\tau}\int_0^\infty m\,e^{-m/\tau}\,dm$$

average salary

社会达成热平衡

$$\frac{1}{\tau} e^{-m/\tau}$$

$$\begin{cases} m: & \text{统计对象的薪水} \\ \tau: & \text{平均薪水} \end{cases}$$

$$\begin{cases} \sum_m p_m = 1 & (1) \\ \sum_m m p_m = \tau & (2) \end{cases}$$

$$\sigma_I = - \sum_m p_m \log p_m \quad (信息熵)$$

$$\sigma^* = \sigma_I + \lambda_1 \left(\sum_m p_m - 1 \right) + \lambda_2 \left(\sum_m m p_m - \tau \right)$$

$$\frac{\partial \sigma^*}{\partial \lambda_1} = 0 \longrightarrow 1^{st} \text{ constraint}$$

$$\frac{\partial \sigma^*}{\partial \lambda_2} = 0 \longrightarrow 2^{nd} \text{ constraint}$$

$$\frac{\partial \sigma^*}{\partial p_m} = 0 \longrightarrow - \log p_m - p_m \cdot \frac{1}{p_m} + \lambda_1 + \lambda_2 m = 0$$

$$\log p_m = \lambda_1 + \lambda_2 m - 1$$

$$p_m = e^{\lambda_1 - 1} e^{\lambda_2 m}$$

$\downarrow$ If only consider 1st cons

all states appear with equal probability

const, can be taken $\frac{1}{Z}$

choose $\lambda_2 \to -\frac{1}{\tau}$

$$p_m \propto e^{-m/\tau}$$

entropy maximization, disorder maximization with the energy constraint

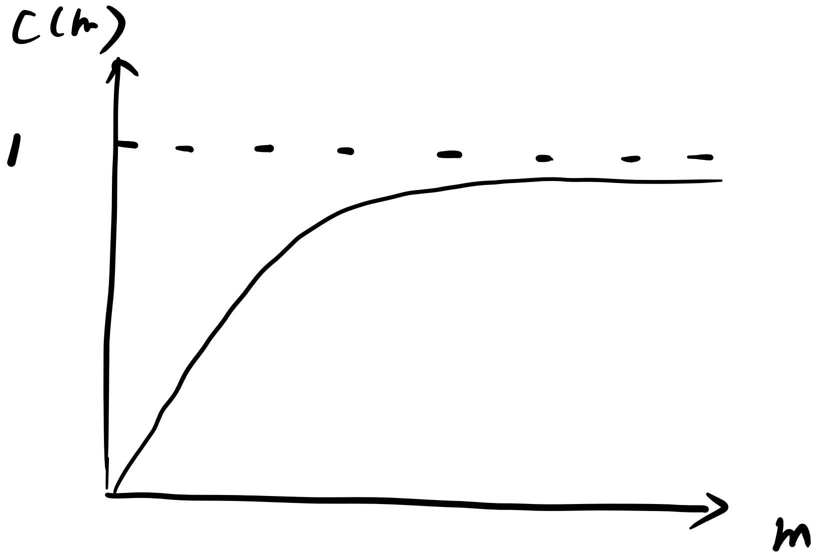
equilibrium for liberty economy

$$C(m) \equiv \int_0^m p(m')\,dm'$$

(收入在 m 以下)

$$\left[\tau = \int_0^\infty e^{-m/\tau}\,dm\right] = \int_0^m \frac{1}{\tau} e^{-m'/\tau}\,dm'$$

$$= 1 - e^{-m/\tau}$$



$$1 - e^{-m_p/\tau} = \frac{1}{10}$$ 前 10% poor

$$m_p = \tau \log\left(\frac{10}{9}\right)$$

$$1 - e^{-m_r/\tau} = \frac{9}{10}$$ 前 10% rich

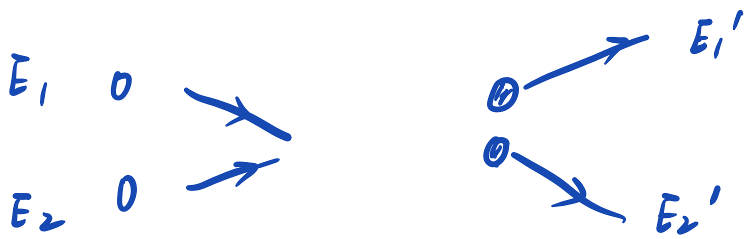
$$m_r = \tau \log 10$$

Disparity ratio $\equiv \dfrac{m_r}{m_p} = \dfrac{\tau \log 10}{\tau \log\left(\frac{10}{9}\right)} = 21.85$ (倍)

Salary : from poor to rich

key physics : microscopic collisions

没有碰撞 → 走不到



Time reversal sym

$$\gamma = \gamma' \quad (\text{rate})$$

$$p(E_1)\, p(E_2)\, \gamma = p(E_1')\, p(E_2')\, \gamma'$$

$$E_1 + E_2 = E_1' + E_2' = E_T$$

$$p(E)\,p(E_T - E) = \text{const}$$

$$\log p(E) + \log p(E_T - E) = \text{const}$$

$$\frac{d\log p}{dE}\Big|_{E} = \frac{d\log p}{dE}\Big|_{E_T - E}$$

$$d\log p / dE = C$$

$$\log p = CE + D$$

$$p(E) = e^{D} e^{CE}$$

Boltzmann distribution

Einstein use the similar way to handle photon $\rightarrow$ planck dist