埃米.诺特　（1882－1935）

----- 20世纪最伟大的数学家

埃米.诺特　是数学界的雅典娜，如果没有她，现代数学和它的教学将会是完全不同的。

---------- 爱因斯坦

Noether Theorem: Every continuous symmetry has a corresponding constant of motion given by the observable that represents the infinitesimal generator of the corresponding transformation.

Nöther theorem

空间平移对称性 ------ 动量守恒

时间平移对称性 ------ 能量守恒

空间转动对称性 ------ 角动量守恒

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物理学中的对称性

①空间坐标平移不变性（系统拉氏函数L不变） $\Rightarrow$ 动量守恒，
雅科比C.G.J.Jacobi（1884）

②L在空间转动下对称 $\Rightarrow$ 角动量守恒，雅科比（1884）

③L在时间平移下对称 $\Rightarrow$ 能量守恒，J.R.Schütz（1897）

④charge conjugation (C)
parity (P)（$\vec{r} \to -\vec{r}$）对称 $\Rightarrow$ 宇称守恒
time reversal (T)（$t \to -t$） $\Rightarrow$ Kramers degeneracy

The **CPT theorem** says that CPT symmetry holds for all physical phenomena,or more precisely, that any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have CPT symmetry. （"mirror-image"）

⑤晶体平移对称性（平移晶格常数 $\vec{a}$ 的整数倍）⇒ Bloch定理

空间平移不变性： 连续→ 离散

⑥全同粒子交换对称性: 玻色子，费米子

⑦标度变换对称性: 临界现象，非线性物理，生命起源……

⑧强相互作用的SU(2)同位旋对称性 (internal dial)

自旋、宇称相同的粒子的内禀对称性。电荷自由度：将中子和质子看成同一粒子的两个不同同位旋状态。

$I_z = 1/2(proton), -1/2(neutron)$ $\binom{p}{n}$ isospin doublet

从数学上说，你能“旋转”质子的同位旋使其变成一个中子，而此时作用于该粒子的强力效应不会改变。

⑨超对称性

Superpartner: the spin difference 1/2

eletron ↔ ‘selectron’（超电子）: a spin-0 particle with the mass of an electron;

photon↔ ‘photino’（光微子）： a spin-1/2 massless particle.

If the photino mirrors the properties of the photon then it must be its own antiparticle： Majorana fermion 。。。
(elusive)

超对称（**supersymmetry**，简称**SUSY**）是费米子和玻色子之间的一种对称性，该对称性至今在自然界中尚未被观测到。

空间平移

- 系统波函数 $\psi(\vec{r})$，系统平移一个矢量 $\vec{\rho}$

以 $P_{\vec{\rho}}$ 表示作用于函数上的平移算符，我们有

$$P_{\vec{\rho}}\psi(\vec{r}) = \psi(\vec{r} - \vec{\rho})$$

选个简单情形，$\vec{\rho} = \rho\vec{e}_x$

$$P_{\vec{\rho}}\psi(\vec{r}) = \psi(\vec{r} - \vec{\rho}) = \psi(x - \rho, y, z)$$

$$P_{\vec{\rho}}\psi(\vec{r}) = \psi(\vec{r} - \vec{\rho}) = \psi(x - \rho, y, z)$$

- 泰勒级数展开

$$\psi(x - \rho, y, z) = \left\{1 - \rho\frac{\partial}{\partial x} + \frac{\rho^2}{2!}\frac{\partial^2}{\partial x^2}\cdots\right\}\psi(x, y, z)$$

$$= \exp(-\rho\frac{\partial}{\partial x})\psi(x, y, z)$$

- 我们利用了 $e^x = 1 + x + \frac{x^2}{2!} + \cdots\frac{x^n}{n!} + \cdots$

- 推广至一般情形 $\psi(\vec{r} - \vec{\rho}) = \exp(-\vec{\rho}\cdot\vec{\nabla})\psi(\vec{r}) = \exp(-\frac{i}{\hbar}\vec{\rho}\cdot\vec{p})\psi(\vec{r})$

$$P_{\vec{\rho}}\psi(\vec{r}) = \psi(\vec{r}-\vec{\rho})$$

$$\psi(\vec{r}-\vec{\rho}) = \exp(-\vec{\rho}\cdot\vec{\nabla})\psi(\vec{r}) = \exp(-\frac{i}{\hbar}\vec{\rho}\cdot\vec{p})\psi(\vec{r})$$

↓

$$P_{\vec{\rho}} = \exp(-\frac{i}{\hbar}\vec{\rho}\cdot\vec{p})$$

$\vec{p} = -i\hbar\vec{\nabla}$：动量算符

幺正算符

波函数满足含时薛定谔方程

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r}) = \hat{H}\psi(\vec{r})$$

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r}) = \hat{H}\psi(\vec{r})$$

以 $P_{\vec{\rho}}$ 作用于上式两边

$$P_{\vec{\rho}}i\hbar\frac{\partial}{\partial t}\psi(\vec{r}) = P_{\vec{\rho}}\hat{H}\psi(\vec{r})$$

问 $P_{\vec{\rho}}\frac{\partial}{\partial t}P_{\vec{\rho}}^{-1} =?$

$$i\hbar\frac{\partial}{\partial t}P_{\vec{\rho}}\psi(\vec{r}) = P_{\vec{\rho}}\hat{H}P_{\vec{\rho}}^{-1}P_{\vec{\rho}}\psi(\vec{r})$$

系统具有空间平移不变性意味着什么？

- 平移后的波函数依然满足薛定谔方程，即

$$i\hbar\frac{\partial}{\partial t}\left[P_{\vec{\rho}}\psi(\vec{r})\right]=\hat{H}\left[P_{\vec{\rho}}\psi(\vec{r})\right]$$

$$\Downarrow$$

$$P_{\vec{\rho}}\hat{H}P_{\vec{\rho}}^{-1}=\hat{H}$$

$$P_{\vec{\rho}}\hat{H}=\hat{H}P_{\vec{\rho}}\Leftrightarrow[P_{\vec{\rho}},\hat{H}]=0$$

$$[\vec{p},\hat{H}]=0$$

动量是守恒量

所有平移算符 $P_{\vec{\rho}}$ 构成群，这个群称为空间平移群

- 连续阿贝尔群

$$P_{\vec{\rho}}P_{\vec{\delta}} = P_{\vec{\delta}}P_{\vec{\rho}} = P_{\vec{\rho}+\vec{\delta}}$$

例1：自由粒子

- 哈密顿量：$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 = \frac{p^2}{2m}$

- 波函数：$\psi(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})$

显然动量守恒 $\psi(\vec{r}-\vec{\rho}) = \exp[i\vec{k}\cdot(\vec{r}-\vec{\rho})]$

也是自由粒子的波函数

例2： 氢原子

- 哈密顿量： $\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{r}$
- 波函数：$\psi(\vec{r}) = R_{nl}(r)Y_l^m(\theta,\varphi)$

动量守恒吗？

- Heisenberg Equation of Motion

  - $U(t, t_0 = 0) = U(t) = e^{-iHt/\hbar}$

  - $A^{(H)}(t) = U^{\dagger}(t)A^{(S)}U(t),\ \psi^{(H)}(t) = \psi^{(H)}(0)$

  - $\frac{dA^{(H)}}{dt} = \frac{1}{i\hbar}\left[A^{(H)}, H\right]$

- Schr$\ddot{o}dinger$ picture:

  - $\bar{A}(t) = \left(\psi(t), A\psi(t)\right)$ with $\psi(t) = U(t)\psi(0)$

  - $\frac{d\bar{A}(t)}{dt} = \frac{1}{i\hbar}\overline{[A, H]} + \overline{\frac{\partial A}{\partial t}}\ \ (\frac{\partial A}{\partial t} = 0,\ not\ explicitly\ include)$

平移：连续→ 离散

布洛赫定理：Bloch theorem

晶格平移群的不可约表示

转动平移算符 $\{R|t\}$

$$x'_i = \sum_j R_{ij} x_j + t_i$$

平移算符 $\{E|t\}$

$t = \vec{R}_l$ 格矢

晶格平移算符 $\{E|\vec{R}_l\}$

$$\vec{R}_l = \vec{R}_1 + \vec{R}_2 + \vec{R}_3 = l_1\vec{a}_1 + l_2\vec{a}_2 + l_3\vec{a}_3,$$

$$l_i \in Z, i = 1,2,3$$

$$\{E|\vec{R}_l\} = \{E|\vec{a}_1\}^{l_1}\{E|\vec{a}_2\}^{l_2}\{E|\vec{a}_3\}^{l_3} \Longrightarrow T = T_1 \otimes T_2 \otimes T_3$$

$\{E|\vec{a}_i\}$及其幂次生成平移群$T_i (i = 1,2,3)$

求不可约表示

- 平移群T_1

选用了周期性边界条件后， x_1方向：N_1个格点
群阶为 N_1 的阿贝尔群：每个群元自成一类

N_1个一维不可约表示，即 N_1 个数

标记表示

用 D^{k_1} 表示群的不可约表示

- 周期性边界条件：$\{E|\vec{a}_1\}^{N_1}=\{E|0\}$

$$D^{k_1}\left(\{E|\vec{a}_1\}^{N_1}\right)=D^{k_1}(\{E|0\})=1$$

$$\left[D^{k_1}(\{E|\vec{a}_1\})\right]^{N_1}=1\Rightarrow D^{k_1}(\{E|\vec{a}_1\})=\exp(-i2\pi p_1/N_1),$$

$$p_1=0,1,2,\ldots,N_1-1$$

即为群元的 N_1 个不可约表示

同理

$$D^{k_2}(\{E|\vec{a}_2\})=\exp(-i2\pi p_2/N_2),$$
$$D^{k_3}(\{E|\vec{a}_3\})=\exp(-i2\pi p_3/N_3),$$

$$p_2=0,1,2,\ldots,N_2-1,$$
$$p_3=0,1,2,\ldots,N_3-1.$$

已知$\mathrm{D}^{\mathrm{k}_1}(\{\mathrm{E}|\vec{\mathrm{a}}_\mathrm{l}\}) = \exp(-i2\pi p_1/N_1), p_1 = 0,1,2,\ldots, N_1 - 1$

$$\mathrm{D}^{\mathrm{k}_1}(\{\mathrm{E}|\mathrm{l}_1\vec{\mathrm{a}}_\mathrm{l}\}) = \exp(-i2\pi l_1 p_1/N_1)$$

综合起来

$$\mathrm{D}^{\mathrm{k}}\left(\{\mathrm{E}|\vec{\mathrm{R}}_\mathrm{l}\}\right) = \mathrm{D}^{\mathrm{k}_1}(\{\mathrm{E}|\mathrm{l}_1\vec{\mathrm{a}}_\mathrm{l}\})\mathrm{D}^{\mathrm{k}_2}(\{\mathrm{E}|\mathrm{l}_2\vec{\mathrm{a}}_2\})\mathrm{D}^{\mathrm{k}_3}(\{\mathrm{E}|\mathrm{l}_3\vec{\mathrm{a}}_3\})$$

$$= \exp\left[-i2\pi\left(\frac{l_1 p_1}{N_1} + \frac{l_2 p_2}{N_2} + \frac{l_3 p_3}{N_3}\right)\right]$$

定义倒格子 $\vec{a}_i \cdot \vec{b}_j = 2\pi\delta_{ij}$

那么倒格矢 $\vec{\mathrm{G}}_{\mathrm{h}} = h_1\vec{b}_1 + h_2\vec{b}_2 + h_3\vec{b}_3,$

$h_i \in Z, i = 1,2,3$

定义

$$\vec{\mathrm{k}}_1 = \frac{p_1}{\mathrm{N}_1}\vec{b}_1, \vec{\mathrm{k}}_2 = \frac{p_2}{\mathrm{N}_2}\vec{b}_2, \vec{\mathrm{k}}_3 = \frac{p_3}{\mathrm{N}_3}\vec{b}_3$$

$$-\frac{\vec{b}_i}{2} \leq \vec{k}_i < \frac{\vec{b}_i}{2}, i = 1,2,3$$

这样

$$\mathrm{D}^{\mathrm{k}_1}(\{\mathrm{E}|\vec{\mathrm{a}}_1\}) = \exp(-i\vec{k}_1 \cdot \vec{a}_1)$$

$$\Rightarrow \mathrm{D}^{\mathrm{k}_1}(\{\mathrm{E}|\mathrm{l}_1\vec{\mathrm{a}}_1\}) = \exp(-i\vec{k}_1 \cdot l_1\vec{a}_1) = \exp(-i\vec{k}_1 \cdot \vec{R}_1)$$

同理

$$D^{k_2}\{E|\vec{R}_2\} = \exp(-i\vec{k}_2 \cdot \vec{R}_2)$$

$$D^{k_3}\{E|\vec{R}_3\} = \exp(-i\vec{k}_3 \cdot \vec{R}_3)$$

$$\vec{R}_i = l_i \vec{a}_i$$

这样，平移群的不可约表示为

$$D^k(\{E|\vec{R}_l\}) = \exp(-i\vec{k} \cdot \vec{R}_l)$$

$$\vec{k} = （\vec{k}_1，\vec{k}_2，\vec{k}_3）$$

观察 $\vec{k} = \vec{k}' + \vec{G}_h$

$$D^k(\{E|\vec{R}_l\}) = \exp(-i\vec{k}\cdot\vec{R}_l) = \exp(-i(\vec{k}' + \vec{G}_h)\cdot\vec{R}_l)$$

已知

$$\vec{G}_h \cdot \vec{R}_l = 2\pi(h_1 l_1 + h_2 l_2 + h_3 l_3) = 2\pi \times (\text{整数})$$

$$D^k(\{E|\vec{R}_l\}) = D^{k'}(\{E|\vec{R}_l\})$$

只需在第一布里渊区选取波矢即可获得平移群 T 的全部不可约表示

$\vec{k}$的个数: $N = N_1 N_2 N_3$

即群的不可约表示的个数

验证不可约表示矩阵元的正交性

$$\sum_{R\in G} \chi^i(R)^* \chi^j(R) = g\delta_{ij}$$

- 均是一维表示，矩阵元即特征标

$$\sum_{\{E|\vec{R}_l\}} D^k(\{E|\vec{R}_l\})\, D^{k'}(\{E|\vec{R}_l\})^*$$

$$= \sum_{l_1}\sum_{l_2}\sum_{l_3} \exp[-i(\vec{k}-\vec{k}')\cdot\vec{R}_l]$$

$$= \sum_{l_1}\sum_{l_2}\sum_{l_3} \exp\left[-i2\pi\left(\frac{l_1(p_1-p_1')}{N_1}+\frac{l_2(p_1-p_1')}{N_2}+\frac{l_3(p_1-p_1')}{N_3}\right)\right]$$

- 关注一项：$\sum_{l_1=0}^{N_1-1} \exp\left[-i2\pi\left(\frac{l_1(p_1-p_1')}{N_1}\right)\right] = \frac{1-\exp[-i2\pi(p_1-p_1')]}{1-\exp[-i2\pi(p_1-p_1')/N_1]}$

整数，
分子为0

当$p_1 = p_1'$时，不一样，等式 $= N_1$，

当$p_1 \neq p_1'$时，等式 $= 0$.

同理，对于其它两个方向

$$\sum_{l_i=0}^{N_i-1} \exp\left[-i2\pi\left(\frac{l_1(p_i - p_i')}{N_i}\right)\right] = N_i\delta_{k_i,k_i'}, i = 2,3$$

- 综合起来

$$\sum_{\{E|\vec{R}_l\}} D^k\left(\{E|\vec{R}_l\}\right) D^{k'}\left(\{E|\vec{R}_l\}\right)^* = N\delta_{k,k'}$$

不可约表示 D^k 的基函数

- D^k 为一维，每一个不可约表示只有一个基函数，以 $\psi_{\vec{k}}(\vec{r})$ 标记

平移群函数变换算符

$$P\{E|\vec{R}_l\}\psi_{\vec{k}}(\vec{r}) = D^k(\{E|\vec{R}_l\})\psi_{\vec{k}}(\vec{r}) = \exp(-i\vec{k}\cdot\vec{R}_l)\psi_{\vec{k}}(\vec{r})$$

又 $\because P\{E|\vec{R}_l\}\psi_{\vec{k}}(\vec{r}) = \psi_{\vec{k}}(\vec{r}-\vec{R}_l)$

$$\exp(-i\vec{k}\cdot\vec{R}_l)\psi_{\vec{k}}(\vec{r}) = \psi_{\vec{k}}(\vec{r}-\vec{R}_l)$$

改写

$$\psi_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})u_{\vec{k}}(\vec{r})$$

周期函数

$$u_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r}-\vec{R}_l)$$

Bloch 定理

时间平移

- 设$\psi(\vec{r},t)$是体系的波函数，仅对时间变量感兴趣，记作 $\psi(t)$

，以 P_τ 表示时间平移操作

$$P_\tau\psi(t) = \psi(t-\tau)$$

泰勒级数展开

$$P_\tau\psi(t) = \exp(-\tau\frac{\partial}{\partial t})\psi(t)$$

$$P_\tau = \exp(-\tau\frac{\partial}{\partial t})$$

$$\text{能量算符}\hat{H} = i\hbar\frac{\partial}{\partial t} \rightarrow P_\tau = \exp(i\tau\hat{H}/\hbar)$$

- 若 $\hat{H}$本身与时间无关， 则有$[P_\tau, \hat{H}] = 0$

$\Rightarrow$ 能量是守恒量

才能使得 $P_\tau \psi(t) = \psi(t - \tau)$

也是方程的一个解 （时间平移不变性所要求）

$\{P_\tau\}$ 也构成连续的阿贝尔群，若成为物理体系的对称群，意味着体系的能量守恒。

- 例： 孤立氢原子

若无微扰，$[P_\tau, \hat{H}] = 0$，当某一时间间隔内原子处于某一特殊状态，那么，原子将持续保持这一状态，总能量也保持不变；

当加入含时微扰，时间平移不变性被破坏，原子会从一个状态跃迁到另一状态，能量不再守恒。

时空-度规 （ metric ）

Lorentz group $SO(3,1)$: parallel discussion with $SO(N)$

Infinitesimal transformations: $L \approx I + i\varphi K$ $\left(R \approx I + i\vec{\theta}\cdot\vec{J}\right)$

$$\eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Keep $(dt \quad dx)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} dt \\ dx \end{pmatrix}$ invariant

$$\begin{pmatrix} dt' \\ dx' \end{pmatrix} = \mathrm{L}\begin{pmatrix} dt \\ dx \end{pmatrix}$$

i.e., $dx'^T \eta dx' = dx^T L^T \eta L dx = dx^T \eta dx \rightarrow L^T \eta L = \eta$

Comparison : $R^T I R = I,$ $L^T \eta L = \eta$

Parallel discussion with $SO(N)$

$$K^T\eta + \eta K = 0$$

$$iK = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$iK_x = \begin{pmatrix} 0 & 1 & & \\ 1 & 0 & & \\ & & & \\ & & & \end{pmatrix} \qquad iK_y = \begin{pmatrix} & & 1 & \\ & & & \\ 1 & & & \\ & & & \end{pmatrix} \qquad iK_z = \begin{pmatrix} & & & 1 \\ & & & \\ & & & \\ 1 & & & \end{pmatrix}$$

Lorentz group $SO(3,1)$的李代数

$(3+1)$ *dimensional spacetime*

R^4上 矢量场的六个生成元

Three rotations iJ

$$-y\partial_x + x\partial_y \equiv iJ_z, \qquad -z\partial_y + y\partial_z \equiv iJ_x, \qquad -x\partial_z + z\partial_x \equiv iJ_y$$

Three boosts iK

$$\frac{1}{c^2}x\partial_t + t\partial_x \equiv iK_x, \qquad \frac{1}{c^2}y\partial_t + t\partial_y \equiv iK_y, \qquad \frac{1}{c^2}z\partial_t + t\partial_z \equiv iK_z$$

keep $c^2(dt)^2 - (dr)^2 = const$

$$[J_i, J_j] = i\boldsymbol{\varepsilon_{ijk}}J_k, \ [J_i, K_j] = i\boldsymbol{\varepsilon_{ijk}}K_k, \ [K_i, K_j] = -\frac{1}{c^2}i\boldsymbol{\varepsilon_{ijk}}J_k$$

$SO(3,1)$ vs $SO(4)$

- $SO(4)$: $J_3 = J_{12}, J_1 = J_{23}, J_2 = J_{31}$ and $K_1 = J_{14}, K_2 = J_{24}, K_3 = J_{34}$

$$[J_i, J_j] = i\boldsymbol{\varepsilon_{ijk}} J_k, \ [J_i, K_j] = i\boldsymbol{\varepsilon_{ijk}} K_k, \ [K_i, K_j] = i\boldsymbol{\varepsilon_{ijk}} J_k$$

- $SO(3,1)$:

$$[J_i, J_j] = i\boldsymbol{\varepsilon_{ijk}} J_k, \ [J_i, K_j] = i\boldsymbol{\varepsilon_{ijk}} K_k, \ [K_i, K_j] = -\frac{1}{c^2} i\boldsymbol{\varepsilon_{ijk}} J_k$$

Digression: $SL(2, C)$ vs $SO(3,1)$

- $SL(2, C)$： all 2-by-2 matrices with complex entries and unit determinant.

for $L \in SL(2, C)$, let $X_M \rightarrow X'_M = L^{\dagger} X_M L$

$$X'_M = x'^0 I - \overrightarrow{x'} \cdot \vec{\sigma}$$
$$\det X'_M = \det X_M \rightarrow$$

$(x'^0)^2 - \vec{x}'^2 = (x^0)^2 - \vec{x}^2$ preserves the Minkowski metric

In fact a Lorentz transformation. This define a map from $SL(2, C)$ to $SO(3,1)$

- $\begin{matrix} L \\ -L \end{matrix} \rightarrow X_M \rightarrow X'_M \, (= L^\dagger X_M L)$

how similar case! $\begin{matrix} u \\ -u \end{matrix} \rightarrow R(u)$

$$SL(2, C)\,/Z_2 = SO(3,1)$$

结合时空平移（Lorentz transformation + translation）

- $x^\mu \to x'^\mu = L^\mu_\nu x^\nu + a^\mu$

⬇

$H = i\frac{\partial}{\partial t}, P_i = -i\frac{\partial}{\partial x_i}$ ($\to P_\mu = i\partial_\mu = i\frac{\partial}{\partial x^\mu}$ (four-component vector))

- $x \to x' = L_2(L_1 x + a_1) + a_2 = L_2 L_1 x + L_2 a_1 + a_2$

群元乘法关系

$$g(L_1, a_1)g(L_2, a_2) = g(L_2 L_1, L_2 a_1 + a_2)$$

结合时空平移 （Lorentz transformation + translation）

$Ten\ generators$: **Poincaré algebra**

⬇

$$\{L, a\} \rightarrow \{J, K, P\}$$

$$[K_i, H] = iP_i, \quad [K_i, P_j] = i\delta_{ij}\frac{1}{c^2}H, \ [J_i, P_j] = i\boldsymbol{\varepsilon_{ijk}}P_k, [P_i, P_j] = 0$$

The conservation of electrical charge

- $\psi \rightarrow \psi' = \psi e^{i\theta}$

Probability explanation:

$$|\psi|^2 = |\psi'|^2$$

What does it bring by θ ?

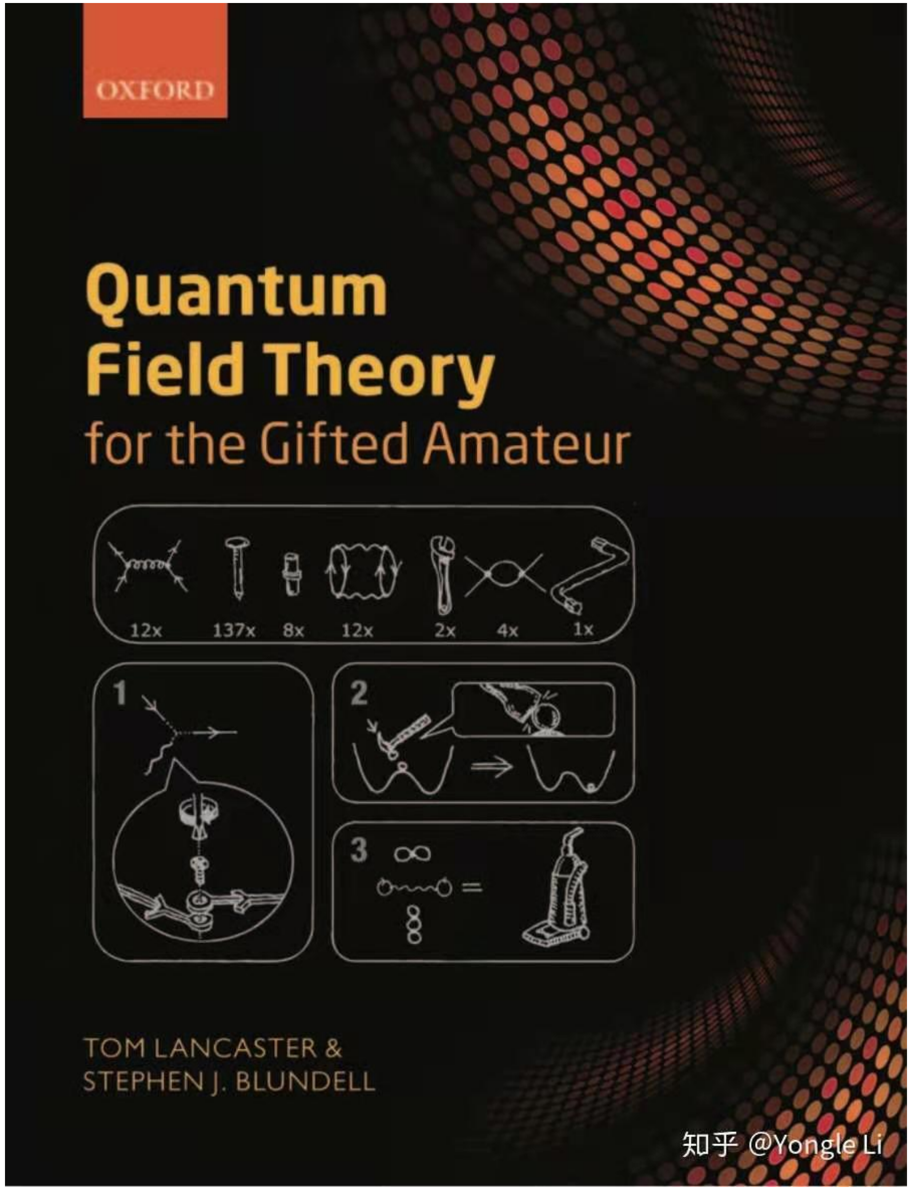
$$\left[\hat{N}, \hat{\theta}\right] = i$$

The conjugate variables:

- $E \quad t$
- $x \quad p$
- $L \quad \theta$
- $N \quad \theta$

From wikipedia

- There are many symmetries in nature besides time translation, such as spatial translation or rotational symmetries. These symmetries can be broken and explain diverse phenomena such as crystals, superconductivity, and the Higgs mechanism.[2] However, it was thought until very recently that time translation symmetry could not be broken.[3] Time crystals, a state of matter first observed in 2017, break time translation symmetry.[4]


OXFORD
Quantum
Field Theory
for the Gifted Amateur
12x
137x
8x
12x
2x
4x
1x
1
2
3
TOM LANCASTER &
STEPHEN J. BLUNDELL
知乎 @Yongle Li

A transformation from one description to another is called a gauge transformation, and the underlying invariance is called a gauge invariance.
Note that gauge invariance is not a symmetry.
Particles do not carry around a knob called 'gauge' that allow us to change them into other particles. Gauge invariance is merely a statement of our inability to find a unique description of a system.

What is a gauge field?

'Gauge' is an awful bit of terminology with which we are unfortunately stuck. Einstein's general relativity showed that spacetime geometry has a dynamical role and Hermann Weyl wondered if the scale of length could itself be dynamical, varying through spacetime. In this picture, one could make a choice of gauge which would be a choice of scale-length: metal wire comes in different thickness or gauges, so the term seemed entirely appropriate. He later adapted his scale argument to one involving phase, as outlined here, but the name 'gauge' stuck.

GREAT

Book !!!