

$N \times N$ matrices

$SU(N)$

$SO(N)$

To solve

$$U^\dagger U = 1,$$
$$\det U = 1$$

$$O^t O = E,$$
$$\det O = 1$$

A.Zee: group theory in a nutshell for physicists

- I.3 Rotations and the Notion of Lie Algebra
- Do what you have to do a little bit at a time
- An infinitesimal angle θ (near the identity)
- $R(\theta) \approx I + A$

Norwegian physicist
Marius Sophus Lie
(1842-1899)

Orthogonal matrix $R^T R = 1$

$$R^T R \approx (I + A)(I + A^T) \approx (I + A + A^T) = I \rightarrow$$

$$A = -A^T \text{ (antisymmetric)} \quad J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \rightarrow A = \theta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$



generator of the rotation group: $R = I + \theta J + O(\theta^2) = \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix} + O(\theta^2)$

• $SO(2)$: $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} = \cos \theta I + \sin \theta J$

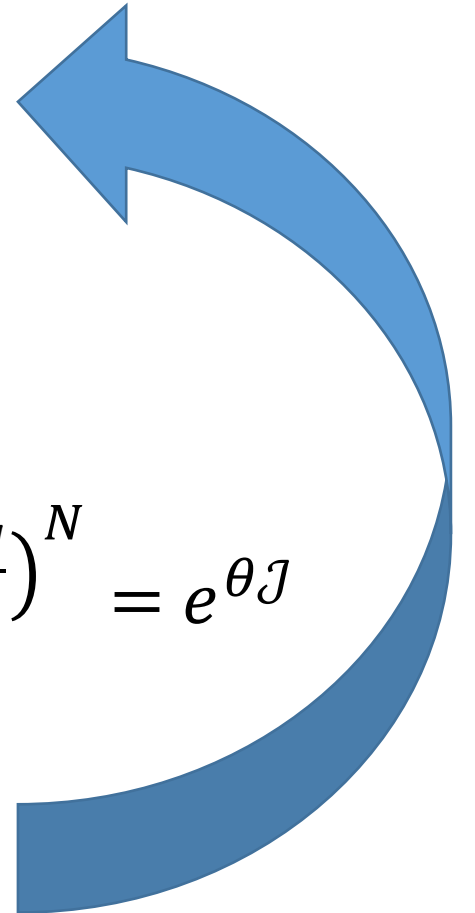


$\theta \rightarrow 0$

$$\begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix}$$

$$R(\theta) = \lim_{N \rightarrow \infty} \left(R\left(\frac{\theta}{N}\right) \right)^N = \lim_{N \rightarrow \infty} \left(1 + \frac{\theta J}{N} \right)^N = \lim_{N \rightarrow \infty} \left(e^{\frac{\theta J}{N}} \right)^N = e^{\theta J}$$

$$e^{\theta J} = \sum_{n=0}^{\infty} \frac{\theta^n J^n}{n!} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$



Two approaches to rotation

1, apply trigonometry, we can get $R(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

2, what is left **invariant**?

$$\vec{u}' = R\vec{u}, \vec{v}' = R\vec{v} \quad \rightarrow \quad \vec{u}'^T \cdot \vec{v}' = \vec{u}^T \cdot \vec{v} \rightarrow R^T R = I$$

Lie: infinitesimal rotation ----- $e^{\theta J} = R(\theta)$

Distance squared between neighboring points

- Check

- $\vec{r}_P = (x, y), \vec{r}_Q = (\tilde{x}, \tilde{y})$

- $\Delta x = \tilde{x} - x, \Delta y = \tilde{y} - y$

- $$\begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

- $\Delta x^2 + \Delta y^2 = \Delta x'^2 + \Delta y'^2 \rightarrow dx^2 + dy^2 = dx'^2 + dy'^2$

To higher-dimensional space

- $ds^2 = \sum_{i=1}^N (dx^i)^2$

$d\vec{x}' = R d\vec{x}$, requirement : ds^2 keep invariant.

$$R^T R = I$$

$\det R = 1$: *eliminate reflection*

- In 3d, 3 generators:

$$\bullet \mathcal{J}_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \quad \mathcal{J}_y = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \mathcal{J}_z = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A = \theta_x \mathcal{J}_x + \theta_y \mathcal{J}_y + \theta_z \mathcal{J}_z$$

$$[J_i, J_j] = i \boldsymbol{\varepsilon}_{ijk} J_k$$

Structure constant

$$R(\theta) = e^A = e^{\sum_i \theta_i \mathcal{J}_i}$$

$$J = -i\mathcal{J} \rightarrow \text{hermitian}$$

$$R(\theta) = e^A = e^{i\vec{\theta} \cdot \vec{J}}: \text{orthogonal matrix}$$



Lie algebra

- $R \approx I + A, R' \approx I + B$: both infinitesimal rotation
- Consider $RR'R^{-1} - R'$: measure the lack of commutativity.
- The difference : $[A, B] \equiv AB - BA$
- $A = i \sum_i \theta_i J_i, \quad B = i \sum_j \theta'_j J_j$

Then $[A, B] = i^2 \sum_{ij} \theta_i \theta'_j [J_i, J_j],$

$[J_i, J_j]^T = -[J_i, J_j],$ antisymmetric, ok, as a linear combination of the J_k

$$[J_i, J_j] = i \boldsymbol{\varepsilon}_{ijk} J_k$$

The **commutation** relations between the generators define a Lie algebra, with $\boldsymbol{\varepsilon}_{ijk}$ referred to as the structure constants of the algebra.

- Lie group: multiplication
- Lie algebra: commutation

As the generators: the antisymmetric matrices

$$C = AB, \quad C^T = B^T A^T = BA \neq -C$$

The algebra does not close under multiplication, only under commutation

In general higher-dimensional space

- $(J_{(mn)})^{ij} = -i(\delta^{mi}\delta^{nj} - \delta^{mj}\delta^{ni})$: generators ($J_{(mn)} = -J_{(nm)}$)
- $J_{(mn)}$: *m*th row, *n*th column: 1 ; *n*th row, *m*th column: -1
elsewhere: 0
- (mn) label matrix (denote which matrix) , *not matrix element* :
i, j: for row and column of $J_{(mn)}$
- $\frac{1}{2}N(N - 1)$ real antisymmetric $N - by - N$ matrices $J_{(mn)}$
m: N options, *n*: $(N - 1)$ options, $\frac{1}{2}$: double counting

$$J_x = J_{23}, \quad J_y = J_{31}, \quad J_z = J_{12}$$

Try it! $(\mathcal{J}_{(mn)})^{ij} = (\delta^{mi} \delta^{nj} - \delta^{mj} \delta^{ni})$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \mathcal{J}_{(12)} \quad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \mathcal{J}_{(23)} \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \mathcal{J}_{(31)}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \mathcal{J}_{(14)} \quad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \mathcal{J}_{(24)} \quad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \mathcal{J}_{(34)}$$

The Lie algebra for $SO(N)$

- $[J_{(mn)}, J_{(pq)}] = i(\delta_{mp}J_{(nq)} + \delta_{nq}J_{(mp)} - \delta_{np}J_{(mq)} - \delta_{mq}J_{(np)})$
- (1) no integer in common, = 0
- (2) one integer in common, refer to $SO(3)$
- (3) two integers in common, = 0

The group $U(N) \rightarrow SU(N)$

- $U^\dagger U = 1, \rightarrow \det(U^\dagger U) = \det(U^\dagger) \det(U) = \det(U)^* \det(U) =$

$$|\det(U)|^2 = 1$$

- $\det(U) = e^{i\alpha}$

S: *special* $\rightarrow \det(U) = 1$: eliminates this dangling phase factor $e^{i\alpha}$

$$U(N) = \begin{cases} e^{i\alpha} I \\ SU(N) \end{cases}$$

Strictly, $U(N) = (SU(N)/Z_N) \otimes U(1)$, $\because \det(e^{\frac{i2\pi k}{N}} I) = 1$, but $e^{\frac{i2\pi k}{N}} I \subset U(1)$

U : 么正, 么模 $\rightarrow M$: 厄米矩阵, 零迹矩阵

$$U \approx I + iM$$

$$(I + iM)^\dagger (I + iM) = I \Rightarrow I + i(M - M^\dagger) + \boxed{M^\dagger M} = I$$

高阶小量

$$M^\dagger = M$$



$$U = e^{iM}, \quad \det U = 1$$

$$\det U = \det e^{iM} = \det e^{iW^\dagger \Lambda W} = \det W^\dagger e^{i\Lambda} W = \det e^{i\Lambda} = \prod_{j=1}^N e^{i\Lambda_j} = e^{i \operatorname{tr} M} = 1$$

M is traceless matrix

$SU(N)$:

独立参数约束条件：

	$M^\dagger = M,$	$trM = 0$
		
	非对角元, $(N^2 - N)/2 * 2$	Traceless : 1
	对角元, N (实数)	

独立参数个数: $2N^2 - (N^2 - N + N + 1) = N^2 - 1$ (减法)

反过来做加法也一样:

对角元 $N - 1$ (实, 零迹) + $(N^2 - N)/2$ (非对角元) * 2 (实虚部) = $N^2 - 1$

$N = 2$, 独立参数个数为3

The structure constants of the Lie algebra

- $U = e^{i\theta^a T^a}$

a : $N^2 - 1$ values

$$[T^a, T^b]^\dagger = [T^{b\dagger}, T^{a\dagger}] = [T^b, T^a] - [T^a, T^b] \text{ (antihermitian)}$$

$$\text{Tr}([T^a, T^b]) = \text{Tr}(T^a T^b) - \text{Tr}(T^b T^a) = 0 \text{ (traceless)}$$

$$\therefore [T^a, T^b] = i f^{abc} T^c$$

- $SU(2)$ 群元的一般形式:

$$u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} = e^{iM}$$

- M 么正, 零迹? Complex matrix

$$N = 2$$

- Hermiticity: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^\dagger = \begin{bmatrix} a^* & c^* \\ b^* & d^* \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = M$
 $b = c^*$, a and d real

- Traceless: $a = -d$

$$\therefore M = \begin{bmatrix} a & b \\ b^* & -a \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \theta_3 & \theta_1 - i\theta_2 \\ \theta_1 + i\theta_2 & -\theta_3 \end{bmatrix}$$

$$M = \frac{1}{2} \vec{\theta} \cdot \vec{\sigma} \quad , \quad \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$$

Pauli matrices here!

$$S = \frac{\sigma}{2} \rightarrow [S_i, S_j] = i\epsilon_{ijk} S_k$$

Question:

- How to check the $SU(2)$ symmetry in a given Hamiltonian?
- How to construct a MPS with $SU(2)$ symmetry?

- M 由零迹的厄米矩阵生成，我们将 $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ 称为 $SU(2)$ 群的无穷小生成元。
- $SU(2)$ 群中任一个与单位元无限接近的元素，可用三个无穷小实参数 $\delta_x, \delta_y, \delta_z$ 匹配生成元生成，即为 $I - i\vec{\delta} \cdot \vec{\sigma}$

对易关系： $[\sigma_x, \sigma_y] = 2i\sigma_z, [\sigma_y, \sigma_z] = 2i\sigma_x, [\sigma_z, \sigma_x] = 2i\sigma_y$

$$S = \frac{\sigma}{2} \rightarrow [S_i, S_j] = i\epsilon_{ijk}S_k$$

- 无穷小生成元 $\sigma_x, \sigma_y, \sigma_z$ 连同它们满足的对易关系，构成李代数 $su(2)$ 。
- $su(2)$ 的任一群元 X 可以写为

$$X = x\sigma_x + y\sigma_y + z\sigma_z = \vec{r} \cdot \vec{\sigma}, \quad x, y, z \in R$$

$SU(2)$ 群在单位元 I 邻域的元素 $I - i\vec{\delta} \cdot \vec{\sigma}$
与该群李代数的元素 $\vec{\delta} \cdot \vec{\sigma}$

有一一对应关系: isomorphic

$$N = 3$$

• $\theta^\alpha \lambda^\alpha (\alpha = 1, 2, \dots, 8)$

Traceless, unitary matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \text{ ---- Gell-mann matrices}$$

We have $\text{tr}(\sigma_a \sigma_b) = 2\delta_{ab}$ and $\text{tr}(\lambda_a \lambda_b) = 2\delta_{ab}$, then $U = e^{i\theta^a \lambda^a / 2}$

$SO(3)$ 群: 行列式为1的三阶实正交矩阵 O , 满足

$$O^t O = I, \det O = 1$$

• 与上述类似, 考虑与单位元 I 无限接近的 $SO(3)$ 群元。

设 M 是无穷小的三阶实矩阵, 即 M 的每个矩阵元都是无穷小的实数, 则与 E 无穷接近的 $SO(3)$ 群元记作 $I + M$ 。由正交条件得

$$M^t = -M$$

三维实空间独立的反对称矩阵可取为

$$J_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \quad J_y = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad J_z = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

• 改成厄米矩阵 $\vec{J} = (J_x, J_y, J_z)$

$$J_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, J_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, J_z = \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$SO(3)$ 群中与单位元无限接近的元素可写为 $I - i\delta\vec{\psi} \cdot \vec{J}$
与其李代数的元素 $\delta\vec{\psi} \cdot \vec{J}$ 一一对应

$SU(2)$ is locally isomorphic to $SO(3)$

• 共同点：三个独立参数

(1) 矩阵 h

(a) 二维迹为0的么正矩阵，是pauli矩阵的线性组合

已知pauli 矩阵

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$h = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = x\sigma_x + y\sigma_y + z\sigma_z = \vec{r} \cdot \vec{\sigma}$$

$$= \begin{bmatrix} z & x - iy \\ x + iy & -z \end{bmatrix}$$

$$h = h^\dagger$$

$$x = \frac{h_{12} + h_{21}}{2}, y = \frac{h_{21} - h_{12}}{2i}, z = h_{11} = -h_{22}$$

(b) 用 u 矩阵对 h 做么正变换

$$h' = uhu^{-1} = \begin{bmatrix} z' & x' - iy' \\ x' + iy' & -z' \end{bmatrix} = \vec{r}' \cdot \vec{\sigma}$$



$$u = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$$



$$u^{-1} = \begin{bmatrix} a^* & -b \\ b^* & a \end{bmatrix}$$

$$h' = uhu^{-1} = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix} \begin{bmatrix} z & x - iy \\ x + iy & -z \end{bmatrix} \begin{bmatrix} a^* & -b \\ b^* & a \end{bmatrix}$$

$$\begin{array}{ccc}
 h = \vec{r} \cdot \vec{\sigma} & \downarrow & h' = uhu^{-1} \\
 h' = \vec{r}' \cdot \vec{\sigma} & & \\
 \vec{r} \rightarrow \vec{r}' & & \vec{r}' = R(u)\vec{r}
 \end{array}$$

每一个使矩阵 h 变成 h' 的么模么正阵 u , 总存在一个 3×3 的矩阵 $R(u)$ 使得 $\vec{r}(x, y, z)$ 变成 $\vec{r}'(x', y', z')$

$$R(u) = \begin{pmatrix} \frac{1}{2}(a^2 + a^{*2} - b^2 - b^{*2}) & \frac{-i}{2}(a^2 - a^{*2} + b^2 - b^{*2}) & -(a^*b^* + ab) \\ \frac{i}{2}(a^2 - a^{*2} - b^2 + b^{*2}) & \frac{1}{2}(a^2 + a^{*2} + b^2 + b^{*2}) & i(a^*b^* - ab) \\ a^*b + ab^* & i(a^*b - ab^*) & aa^* - bb^* \end{pmatrix}$$

- (2) $R(u)$ 的性质
 - (a) $R(u)$ 是个实矩阵: r'_i, r_i 都是实数

$$r'_i = \sum_j R(u)_{ij} r_j$$

- (b) $R(u)$ 是转动矩阵:

么正变换不改变矩阵的行列式

$$\det h' = \det u \cdot \det h \cdot \det u^{-1} = \det h$$

$$\det h = -z^2 - (x + iy)(x - iy) = -(x^2 + y^2 + z^2)$$

$$\det h' = -(x'^2 + y'^2 + z'^2)$$

$$x'^2 + y'^2 + z'^2 = x^2 + y^2 + z^2$$



- (c) $\det R(u) = 1$

转动矩阵的行列式为1或-1，这里所有的转动矩阵，可由矩阵 u 的参量 (a, b) 连续变化而得，



$$(1, 0) \rightarrow u = I_0 \rightarrow \det R(I_0) = 1$$

行列式不能发生由1到-1的跳变，对于所有参量 (a, b) 都有

$$\det R(u) = 1$$

$R(u)$: 三维空间的正当转动

$SU(2)$ covers $SO(3)$ twice

- 零迹矩阵 h 作为媒介

$$\begin{aligned} h' &= u h u^{-1} \\ \vec{r}' &= R(u) \vec{r} \end{aligned} \quad \downarrow$$

- but

$$\begin{aligned} h' &= (-u) h (-u)^{-1} \\ \vec{r}' &= R(u) \vec{r} \end{aligned} \quad \downarrow$$

$$\begin{array}{l} u \\ -u \end{array} \begin{array}{l} \rightarrow \\ \rightarrow \end{array} R(u)$$

2:1 的同态关系

Identity

$$\begin{aligned} \bullet \exp\left(\frac{i\vec{\sigma}\cdot\vec{n}\alpha}{2}\right) &= \left[1 - \frac{(\vec{\sigma}\cdot\vec{n})^2}{2!} \left(\frac{\alpha}{2}\right)^2 + \frac{(\vec{\sigma}\cdot\vec{n})^4}{4!} \left(\frac{\alpha}{2}\right)^4 - \dots \right] \\ &\quad + i \left[(\vec{\sigma}\cdot\vec{n}) \frac{\alpha}{2} - \frac{(\vec{\sigma}\cdot\vec{n})^3}{3!} \left(\frac{\alpha}{2}\right)^3 + \dots \right] \\ &= \mathbf{1} \cos\left(\frac{\alpha}{2}\right) + i (\vec{\sigma}\cdot\vec{n}) \sin\left(\frac{\alpha}{2}\right) \end{aligned}$$

$$(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b} + i\vec{\sigma}\cdot(\vec{a}\times\vec{b})$$

$$(\vec{\sigma}\cdot\vec{n})^n = \begin{cases} 1 & \text{for } n \text{ even} \\ \vec{\sigma}\cdot\vec{n} & \text{for } n \text{ odd} \end{cases}$$

You can check

$$U = \exp\left(\frac{i\vec{\sigma}\cdot\vec{n}\alpha}{2}\right)$$

$U(2\pi) = -I, U(4\pi) = I$: *double covering, only locally isomorphic*

- $U^\dagger\sigma_1U = \cos\alpha\sigma_1 + \sin\alpha\sigma_2$
- $U^\dagger\sigma_2U = -\sin\alpha\sigma_1 + \cos\alpha\sigma_2$
- $U^\dagger\sigma_3U = \sigma_3$

Half angles come in to render the Lie algebra of $SU(2)$ and $SO(3)$
the same !

关于算符与波函数

- 对于自旋pauli算符支撑起的零迹空间，
测量算符由 $\hat{\sigma}_z = \vec{r} \cdot \vec{\sigma}$ ， $\vec{r} = (0,0,1)$ 。 \vec{r} 经过
旋转变换（绕着y轴转 θ 角）而得

$$\vec{r}' = (\sin \theta, 0, \cos \theta)$$

$$\hat{O} = \vec{r}' \cdot \vec{\sigma} = \sin \theta \hat{\sigma}_x + \cos \theta \hat{\sigma}_z$$

求其本征态 $|+\rangle$ ，满足 $\hat{O}|+\rangle = |+\rangle$

对应关系

$$\vec{r} = (0,0,1) \quad \longrightarrow \quad \vec{r}' = (\sin \theta, 0, \cos \theta)$$

$$|\uparrow\rangle \quad \longrightarrow \quad |+\rangle = \cos \frac{\theta}{2} |\uparrow\rangle + \sin \frac{\theta}{2} |\downarrow\rangle$$

- 三维空间中 \vec{r} 矢量的 θ 角旋转，
- 自旋空间中本征矢 $|\uparrow\rangle$ 的 $\theta/2$ 角旋转。

- Group is different, algebra is the same
- Locally isomorphic

$$N^2 - 1 = \frac{M(M - 1)}{2} \rightarrow$$

- $SU(2)$ vs $SO(3)$

$$N = 2, M = 3$$

- $SU(4)$ vs $SO(6)$

$$N = 4, M = 6$$

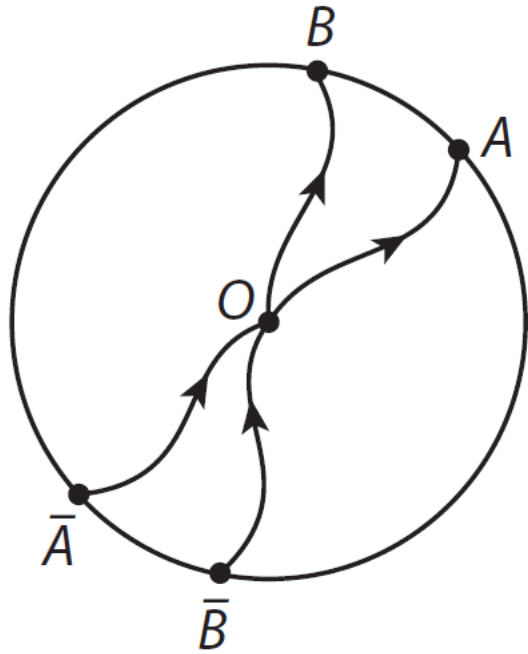
Manifold of Lie group $\pi_1 (M)$

The (first) homotopy(同伦) group $\pi_1 (M)$ of a manifold M

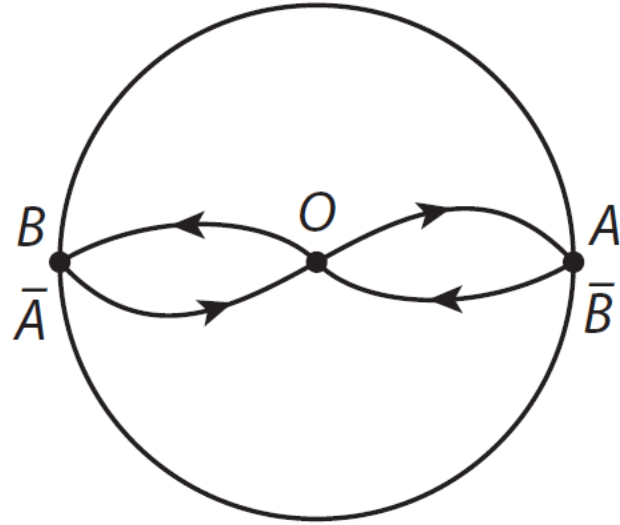
Two closed curves in a manifold are said to be homotopic to each other if they can be continuously deformed into each other.

$SO(3)$

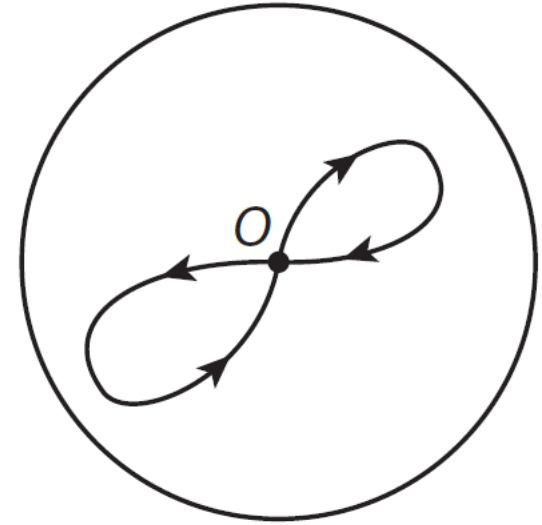
• $\pi_1(SO(3)) = Z_2$ (双连通)



(a)



(b)



(c)

$$R(\vec{\omega}, \alpha) = R(-\vec{\omega}, 2\pi - \alpha)$$

$\alpha = \pi$: the same element \rightarrow one curve one jump

With even jumps, can continuous deform to one point

odd jumps, can not \sim

- $\pi_1(SU(2)) = \emptyset$

$$u(\vec{\omega}, \alpha) = -u(-\vec{\omega}, 2\pi - \alpha)$$

$\alpha = \pi$: different elements : the two points connected by the origin in the sphere

- This topological consideration confirms what we already know, that $SU(2)$ is locally isomorphic to $SO(3)$, but since $SU(2)$ double covers $SO(3)$, they cannot possibly be globally isomorphic.

Group Theory in a Nutshell for Physicists



A. Zee

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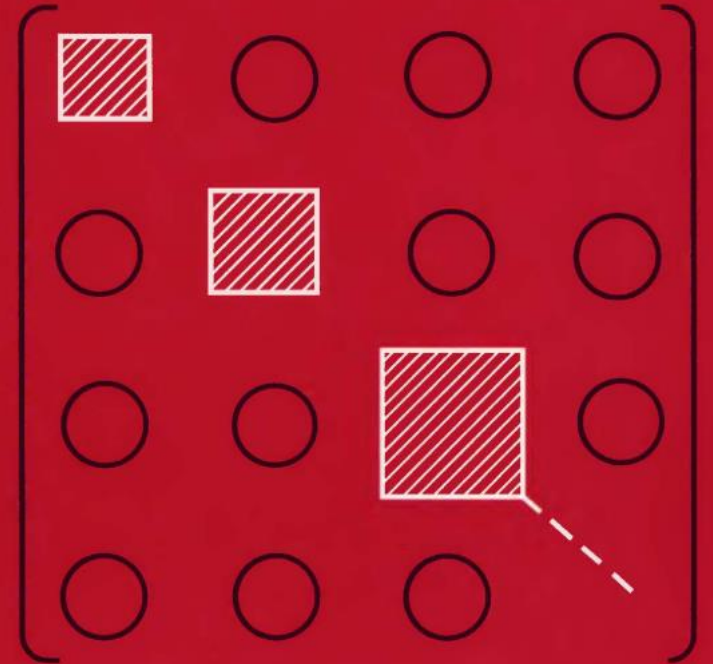
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