Tensors: $i, j = 1, 2, \dots, N$ in the N-dim space

T^{ij} : N^2 mathematical entities

that transform into linear combinations of one another

E.g. $N = 3$

9 entities

$T^{11}, T^{12}, T^{13}, T^{21}, \dots, T^{33}$

$$T'^{21} = R^{2k} R^{1l} T^{kl} = \text{sum of } 9 \text{ terms} \quad \sum_k \sum_l$$

R is the rotation in N-dim space.

$R^T R = I$

$\det R = 1$

A vector is defined by how it transforms under a rotation.

$$V^i \to V'^i = R^{ij} V^j$$

with $i, j = 1, 2, \cdots, N$

$$T^{ij} \rightarrow T'^{ij} = R^{ik} R^{jl} T^{kl}$$

$$T' = \left(D(R)_{(ij),(kl)} = R^{ik} R^{jl} \right) \begin{pmatrix} T^{11} \\ T^{12} \\ T^{13} \\ \vdots \\ T^{33} \end{pmatrix}$$

$$V_i' = R_{ij} V_j$$

each of two indices on T^{ij} transforms independently, that is, in parallel without interfering with each other